Probability Distribution Characterization of Link Parameters in LEO Constellations Over Brazil

Guilherme da Silva E. Ferreira and Alberto G. Guimarães

***Abstract*—This paper presents a study on the probability distribution of key link parameters in low Earth orbit (LEO) satellite systems, focusing on terminals located in Brazil. A large dataset of satellite orbital trajectories from four different operators is generated through simulation, and the probability density functions (PDF) of relevant link parameters are then estimated using either known analytical functions or by fitting histogram data through polynomial regression. The results presented herein provide simple and accurate statistical models, and a quantile-based goodness-of-fit metric is used to evaluate the agreement between the empirical data and the proposed distributions. Although this type of study has been explored in recent literature, there is still a lack of evaluations specifically addressing coverage across Brazilian territory.**

***Index Terms*—LEO satellite constellation, satellite communications, probability distribution estimation.**

I. Introduction

Low Earth Orbit (LEO) satellite constellations have been used in telecommunication systems since the 1990s, initially focusing on voice and data services with low data rates. However, the last five years have marked a turning point for the importance of this architecture in the satellite telecommunications landscape, with the consolidation of traditional LEO satellite operators (*e.g.* Iridium and Globalstar) and the emergence of new and relevant players in this market (*e.g.* Starlink, Amazon LEO, OneWeb, AST SpaceMobile etc.) [1].

Compared to geostationary orbit, the shorter distance of LEO orbits to Earth allows connections with lower delay and attenuation; however, this advantage is accompanied by technological challenges for the link. The need for handovers between satellites in the constellation, a received signal with significant Doppler shift, and the use of more elaborate network access and scheduling methods are some of the problems considered in systems that employ the LEO orbit [1], [2].

To address such challenges, it is necessary to have a proper understanding of the parameters associated with this type of orbit, as they impact the design and management of the connection. In geostationary satellite links, many of these parameters are theoretically constant; however, for LEO orbits, they exhibit significant variation and are usually modeled as random variables for stochastic characterization [3]–[7].

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Recent works in the literature perform statistical analyses of orbital geometry by fitting Probability Density Function (PDF) curves to histograms from simulated orbits (an empirical approach) [7], or, in some cases, by mathematically deriving PDF expressions (an analytical approach) [4]–[6].

However, directly applying the results from these studies to specific scenarios (for instance, a connection from a given constellation to a terminal in Brazil) presents challenges. This is because, in the empirical approach, results are obtained for a particular scenario and may not be generalizable to others; and in the analytical approach, typically the PDF expressions either depend on orbital parameters that are difficult to assess (*e.g.* in [5] the central angle between the satellite's ground projection and the center of the cell where the terminals are located must be available) or are developed under different modeling frameworks for the parameter of interest [6].

In this work, an extensive survey of link parameter values for four relevant commercial LEO constellations (Starlink, Globalstar, Iridium, and OneWeb) is carried out via simulation, considering Earth terminals located in the northern and southern regions of Brazilian territory. With the large dataset obtained from this survey, PDF estimates are made for the parameters *Elevation Angle*, *Doppler Shift*, and *Contact Time*. This involves considering, where possible, parametric modeling based on classical PDFs, and non-parametric modeling when the data do not lend themselves to estimation using known functions or mixtures of known PDFs. The obtained results provide an important set of information for the implementation and performance studies of LEO links within the Brazilian territory.

The remainder of this paper is organized as follows: Section II defines the scenario considered in this study, and Section III presents the obtained results and their corresponding analysis. Section IV presents the conclusions of the work.

II. Scenario under Analysis

A. LEO Satellite Constellations

LEO satellite constellations used in telecommunication networks can be classified into two categories, according to the performance and applications they provide on their links: i) systems launched over three decades ago and improved over time, but designed for low-data-rate services (voice, IoT, messaging, etc); and ii) systems deployed within the last decade that enable broadband applications. In the first category, two major representatives are the Iridium and Globalstar systems, whereas in the second, the Starlink and OneWeb constellations are very well-established systems. These four systems operate in Brazil and serve as the basis for the present study.

B. Ground Station Locations

Two cities in Brazil were selected as ground station locations, representing the southern and northern part of the country: Petrópolis, RJ (22°27'09.6"S , 43°14'27.7"W) and São Gabriel da Cachoeira, AM (0°28'39.3"S , 67°20'36.4"W).

C. Link Parameters Assessed

1) Elevation Angle: It is the angle between the *terminal-satellite* line and the horizon line established at the terminal's location. Its value influences signal attenuation from nearby obstructions, the amount of multipath, atmospheric effects on the signal, and interference from terrestrial signals [8].

2) Doppler Frequency Shift: Links in LEO systems are subject to a significant Doppler frequency shift in the received signal, which degrades the connection's performance, particularly for OFDM-based waveforms which are especially sensitive to frequency deviations [9]. On the other hand, the statistical characterization of this parameter for a given constellation, allows for the creation of mechanisms to mitigate its deleterious effects on the link.

The Doppler shift, *normalized* with respect to the carrier frequency, is given by $\nu = f_d/f_c = -v/c$ [8] , where f_d is the Doppler shift, f_c is the carrier frequency, c is the speed of light, and v is the radial velocity between the terminal and the satellite, which can be calculated as the time derivative of the distance (r) between them, *i.e.*, $v = dr/dt$.

3) Contact time: This parameter refers to the time interval during which a satellite remains above the ground station's minimum elevation angle. In other words, it is the time window in which communication is possible during a satellite pass. This is a fundamental metric for the planning and operation of the communication system [4], and the statistical analysis of this parameter is, therefore, crucial for constellation design, as it helps define the number of satellites required to meet a coverage and link performance requirement [3], [7].

III. RESULTS AND ANALYSIS

The characterization of the probability distribution for the parameters mentioned in Section II was performed based on extensive orbit simulations of the four aforementioned constellations. For this task, recently released two-line element (TLE) files for the constellation satellites were obtained, and the *Simplified General Perturbations Model (SGP4)* [10] was applied. Values for orbital positions and parameter calculations were obtained at 10-second intervals over a total observation period of approximately 60 days, considering at least 4 satellites belonging to each constellation.

This procedure yielded a large dataset for each system-location pair, and the distributions of the parameters of interest are graphically represented by the histograms shown in Figs. 1 to 3. Continuous red lines were plotted over the histograms, following specific PDF expressions with estimated parameters (for elevation angle and contact time PDF characterization), or a polynomial regression model (for Doppler shift PDF characterization) based on the estimated histogram.

The agreement between empirical data and theoretical distributions was quantified using the Mean Absolute Error of Quantiles (MAEQ) defined below, following a quantile-based goodness-of-fit approach:

$$MAEQ = \frac{1}{19}\sum_{i=1}^{19}\left|\frac{Q_{emp}(p_i) - Q_{model}(p_i)}{Q_{emp}(p_i)}\right| , \qquad (1)$$

where $Q_{emp}(p_i)$ and $Q_{model}(p_i)$ are respectively the quantiles values of the empirical distribution and the proposed pdf model, and $p_i = 5\%, 10\%, \ldots, 95\%$ is the probability level.

Figs. 1(a) to 1(h) present the normalized histograms and the corresponding fitted curves for the *elevation angle* parameter. From the set of values obtained over the 60-day period, only angle values greater than 20° were considered for the histogram estimation, as this is the minimum value that most operators consider as operational for the links [1]. The histograms shape indicates that the *shifted and truncated* decreasing exponential function [11] is an appropriate choice to model the PDF in all scenarios, whose definition is

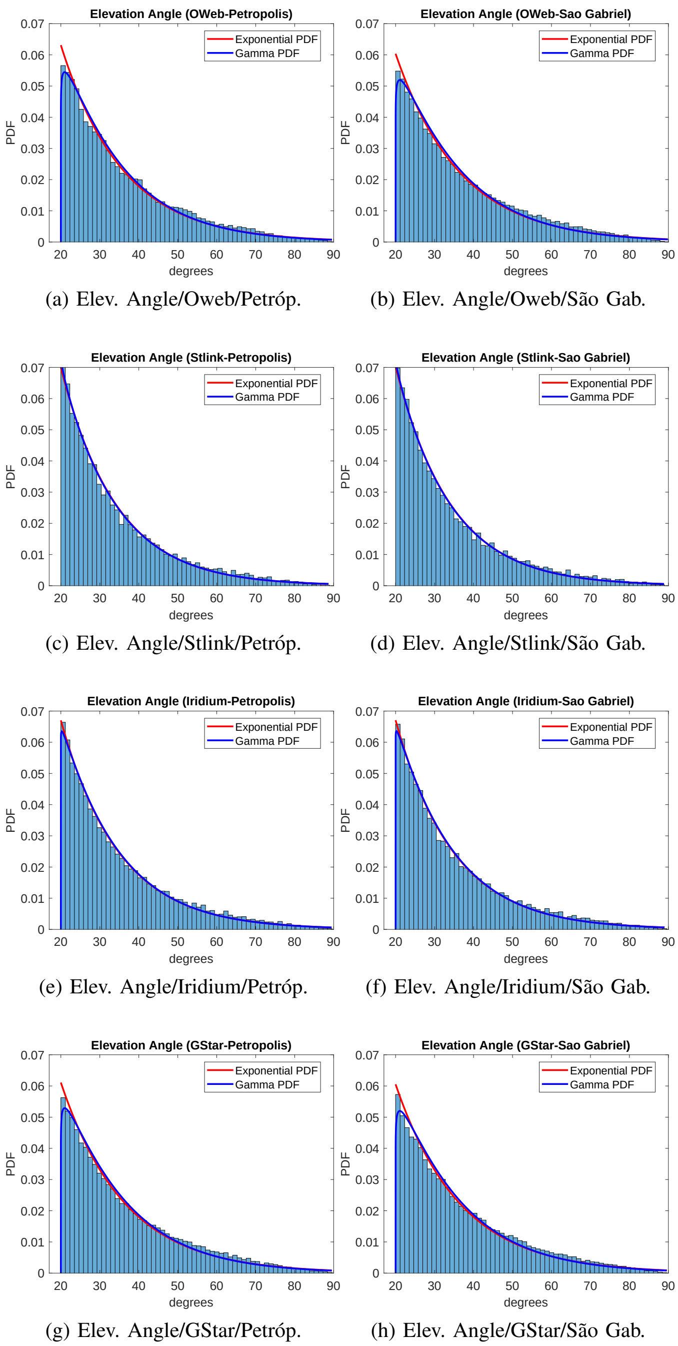
$$f_{el-angle}(\theta) = \frac{\lambda e^{-\lambda(\theta-20)}}{1-e^{-70\lambda}} , \quad 20 \le \theta \le 90, \qquad (2)$$

and $f_{el-angle}(\theta) = 0$ otherwise. The rate parameter (λ) in (2) is estimated for each scenario using the Maximum-Likelihood (ML) method [12], assuming that the data are independent and identically distributed, yielding $\hat{\lambda}_{ML} = (\bar{\theta} - 20)^{-1}$, where $\bar{\theta}$ is the sample mean of the corresponding dataset. The estimates are shown in the third column of Tab. I and these values indicate that the terminal location does not significantly affect the probability distribution, but slight differences can be observed among satellite operators. The values lie in the range of 0.06 to 0.07, and the corresponding difference for the mean value of the elevation angle ($\approx 20^\circ + \lambda^{-1}$) is about 2.4°. The difference in the mean elevation angle can be mainly attributed to the differences in their orbital altitudes (Globalstar and Oneweb operate at higher altitudes, while Iridium and Starlink satellites are in lower orbits [8]). The MAEQ values, also presented in Tab. I, confirm that the exponential PDF provides an excellent fit to the observed data.

Table I: Results for *elevation angle* PDF estimation.

System	**City**	λ (deg^{-1})	$MAEQ$ (%)
Starlink	Petrópolis	0.070	0.96
Starlink	São Gabriel	0.070	0.91
OneWeb	Petrópolis	0.063	1.67
OneWeb	São Gabriel	0.061	1.97
Iridium	Petrópolis	0.067	0.98
Iridium	São Gabriel	0.067	0.89
Globalstar	Petrópolis	0.061	1.92
Globalstar	São Gabriel	0.060	2.05

Fig. 1 also presents a histogram fitting by Gamma PDF curves, whose *shape* and *rate* parameters were estimated using the ML method as well. Gangorra et al., in [3] and [7], assumed this model to characterize the elevation angle in generic LEO link scenarios. A visual comparison of the exponential and Gamma curves in Fig. 1, together with the $MAEQ$ values obtained for both models, indicates that their fitting performances are practically indistinguishable. It is worth noting that the exponential distribution is a special case of the Gamma distribution when the *shape* parameter equals one. Indeed, the estimated *shape* parameter values in our study are very close to unity, ranging from 0.987 to 1.07. Therefore, the hypothesis assumed in this work is that the elevation angle, for LEO links in Brazil, is appropriately characterized by the exponential PDF, given by expression (2). This choice provides a simpler model while maintaining essentially the same fitting accuracy as the more general Gamma distribution.



(a) Elev. Angle/Oweb/Petróp. (b) Elev. Angle/Oweb/São Gab.

(c) Elev. Angle/Stlink/Petróp. (d) Elev. Angle/Stlink/São Gab.

(e) Elev. Angle/Iridium/Petróp. (f) Elev. Angle/Iridium/São Gab.

(g) Elev. Angle/GStar/Petróp. (h) Elev. Angle/GStar/São Gab.

Figure 1: Histograms and PDF estimation of the elevation angle.

The estimated histograms of the *normalized Doppler shift* (Hz/MHz) are presented in Figs. 2(a) to 2(h). The distributions are symmetric around zero, exhibiting a "U" shape with maximum values at the extremes; some histograms also show local maxima around zero, although these are not significant. The Doppler shift is contained within the range $[-20, 20]$ Hz/MHz for Oneweb and Globalstar, while extending over a slightly wider interval for Starlink and Iridium due to their higher satellite orbital speeds. To our knowledge, no "classical" PDF adequately represents the distributions in Fig. 2; therefore, an analytical representation based on polynomial regression was adopted. An 8th-order regression was fitted to the histogram values and normalized to unit area, yielding the curves shown in Fig. 2. The good agreement between the fitted curves and histograms is visually evident and supported by the low $MAEQ$ values in Tab. II. Finally, Oneweb shows a noticeable difference between the two locations, with greater probability mass at the interval extremes in São Gabriel, indicating more severe Doppler shift conditions in this region.

Table II: $MAEQ$ values for *Doppler shift* histogram fitting.

System	**City**	$MAEQ$ (%)
Starlink	Petrópolis	1.59
Starlink	São Gabriel	0.98
OneWeb	Petrópolis	0.47
OneWeb	São Gabriel	0.92
Iridium	Petrópolis	0.53
Iridium	São Gabriel	1.30
Globalstar	Petrópolis	0.62
Globalstar	São Gabriel	0.68

The estimated histograms for the *contact time* are presented in Figs. 3(a) to 3(h). A model that seems suitable for representing the contact time statistics of the OneWeb and Starlink constellations is an *increasing and truncated* exponential [11], whose PDF expression is

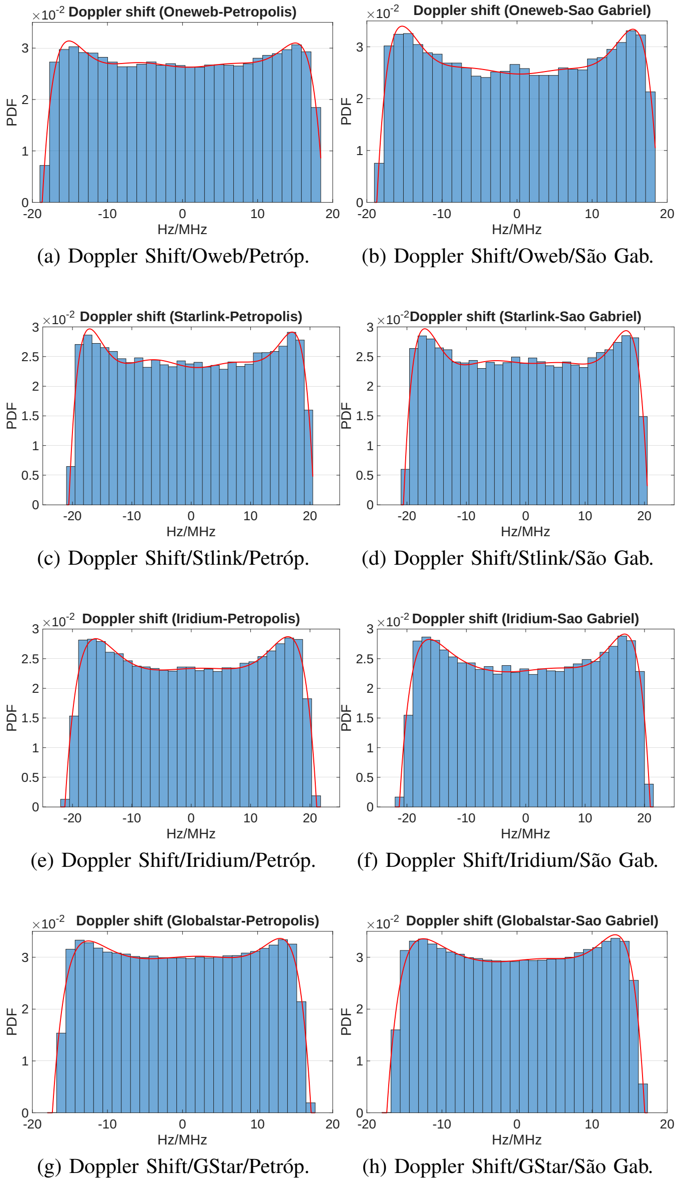
$$f_{cnt-time}(t) = \frac{\lambda e^{\lambda t}}{e^{\lambda c} - 1} \ , \ \ 0 \leq t \leq c \ , \tag{3}$$

vanishing for $t \notin [0, c]$. The parameter c is straightforwardly obtained from the data set, *i.e.*, it is the maximum value observed in the simulation, and λ is estimated using the ML method. The values are presented in Tab. III, together with the $MAEQ$ values, which indicate a high degree of agreement between the proposed PDF and the observed data distribution.

In turn, the contact time histograms obtained for the Iridium and Globalstar satellites (Figs. 3(e)-(h)) seem to exhibit at least one mode (*i.e.* a density peak). An appropriate model to describe the probability distribution behavior in this case is a mixture of two Gaussian distributions and a increasing exponential distribution [11] expressed as

$$f_{cnt-time}(t) = \pi_E \frac{\lambda e^{\lambda t}}{e^{\lambda c} - 1} + \sum_{i=1}^{2} \pi_{G,i} \mathcal{N}(t; \mu_i, \sigma_i) \ , \tag{4}$$

holding in the interval $[0, c]$ and vanishing otherwise, where $\mathcal{N}(t; \mu_i, \sigma_i)$ denotes a truncated Gaussian PDF with parameters (μ_i, σ_i), and $\pi_{G,i}$ $(i = 1, 2)$ and π_E are the mixture weights associated to the Gaussian and exponential PDFs, respectively. The Gaussian and exponential parameters and their weights are estimated using the iterative ML-EM (Maximum Likelihood-Expectation Maximization) method [12], yielding the values in Tab. IV. The MAEQ values indicate good agreement between the empirical data and proposed distributions. The results show that: i) Globalstar data are



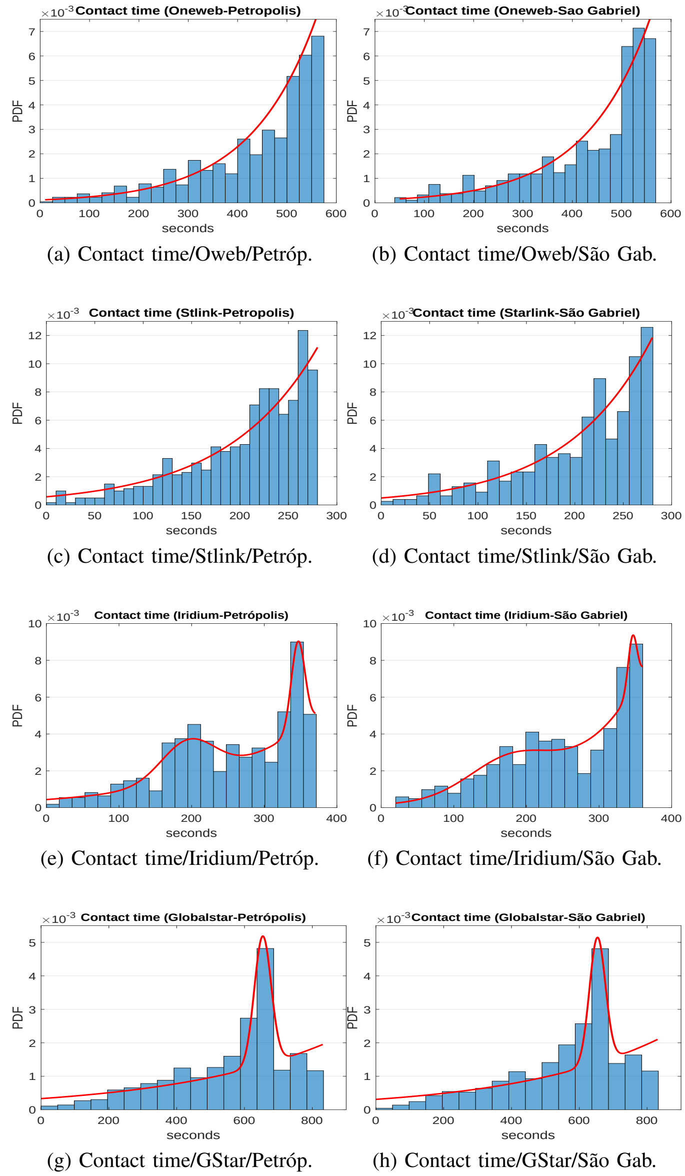
(a) Doppler Shift/Oweb/Petróp. (b) Doppler Shift/Oweb/São Gab.

(c) Doppler Shift/Stlink/Petróp. (d) Doppler Shift/Stlink/São Gab.

(e) Doppler Shift/Iridium/Petróp. (f) Doppler Shift/Iridium/São Gab.

(g) Doppler Shift/GStar/Petróp. (h) Doppler Shift/GStar/São Gab.

Figure 2: Histograms and polynomial regression fit for the normalized Doppler shift.



(a) Contact time/Oweb/Petróp. (b) Contact time/Oweb/São Gab.

(c) Contact time/Stlink/Petróp. (d) Contact time/Stlink/São Gab.

(e) Contact time/Iridium/Petróp. (f) Contact time/Iridium/São Gab.

(g) Contact time/GStar/Petróp. (h) Contact time/GStar/São Gab.

Figure 3: Histograms and PDF estimation of the contact time.

adequately fitted by the exponential component and a single Gaussian, as the second Gaussian has negligible weight; ii) the exponential component has a significant contribution (*i.e.*, high weight) to the probability distributions of both systems; and iii) Globalstar model parameters are remarkably similar for the two locations, suggesting that this PDF characterization is valid throughout Brazil. This is not observed for Iridium.

Table III: Estimated parameters and $MAEQ$ values for the *contact time* PDF considering OneWeb and Starlink systems.

	OneWeb		**Starlink**	
Parameter	Petróp.	S. Gabriel	Petróp.	S. Gabriel
c (s)	570	570	285	285
λ (s^{-1})	0.0074	0.0075	0.0106	0.0113
$MAEQ$ (%)	3.44	3.83	2.95	1.63

Table IV: Estimated parameters and $MAEQ$ values for the *contact time* PDF considering Iridium and Globalstar systems.

	Iridium		**Globalstar**	
Parameter	Petróp.	S. Gabriel	Petróp.	S. Gabriel
c (s)	370	360	840	840
π_{G1}	0.199	0.268	0.239	0.221
μ_1 (s)	195.19	178.84	653.66	653.31
σ_1 (s)	37.19	56.65	24.83	23.57
π_{G2}	0.113	0.048	–	–
μ_2 (s)	346.48	346.00	–	–
σ_2 (s)	9.40	6.31	–	–
π_E	0.687	0.689	0.761	0.778
λ (s^{-1})	0.0065	0.0102	0.0021	0.0023
$MAEQ$ (%)	1.30	1.68	6.03	6.33

IV. Conclusions

This paper presented an empirical survey through simulation of the probability distributions of the *elevation angle*, *Doppler shift*, and *contact time* for links between LEO satellite constellations and user terminals located in the northern and southern regions of Brazil. Accurate probability density function (PDF) models were then obtained through either classical distributions or polynomial regression, as confirmed by both visual inspection and numerical comparisons based on distribution quantiles. It is observed that the elevation angle follows a shifted decreasing exponential distribution (a simpler yet equally accurate alternative to the Gamma distribution reported in [7]), the Doppler shift has a bimodal distribution with maxima at the boundaries of its support interval, and the contact time exhibits distinct behaviors among the constellations: for OneWeb and Starlink, the distribution is well-modeled by an increasing exponential function, whereas for Iridium and Globalstar, whose histograms display one or more modes, accurate modeling requires mixture models combining Gaussian components with an increasing exponential term. It is also noticed that the terminal's location has impact on the probability distribution of the Doppler shift and contact time parameters for some constellations.

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