

Partial Update NLMS Algorithms with varying Time-updating Coefficients for Channel Identification and Echo Cancellation

Tadeu N. Ferreira, Markus V. S. Lima, Marcello L. R. de Campos, Paulo S. R. Diniz

Abstract—In this paper, partial update adaptive algorithms with a time-varying number of updating coefficients are proposed for channel identification and echo cancellation applications. The proposed algorithms present lower computational complexity than Partial Update NLMS M Maxima and a faster convergence than the Sparse Partial Update algorithm. The proposed algorithms allow a reduction of updates with hardly increasing the mean-square error in comparison to the benchmark algorithms. Sensitivity analysis to the proposed thresholds shows that a reduction of computational complexity may be reached with low MSE degradation.

Index Terms—adaptive filter, communications, partial update, system identification

I. INTRODUCTION

Iterative statistical filters with signal processing applications have led to the development of adaptive filtering [1]. Among several configurations, system identification is one of the most common scenarios of adaptive filters. Adaptive filters for system identification include applications such as echo cancellation and channel identification, which demand fast processing, and thus require low-complexity algorithms.

The Partial update (PU) techniques reduce the overall computational complexity of adaptive filtering while providing a good trade-off with the mean square error (MSE) [1], [2]. The initial PU algorithms' update follows a round-robin rule among

the coefficients, such as the Sequential Partial Update Normalized Least-Mean Squares (SPNLMS) algorithm [3], whose convergence is considerably decreased by skipping the coefficient updates.

The Partial Update NLMS M Maxima (PU NLMS M -max) [4] algorithm ranks the coefficient elements using the magnitude of the corresponding gradient estimates, then PU NLMS M -max updates M coefficients, by choosing the indices corresponding to the highest gradient elements. In general, PU NLMS M -max mitigates the decrease in the speed of convergence of the PU adaptive filter. In [5], the Sparse Partial Update (SPU) NLMS algorithm provides an update with a time-varying threshold based on the variance of the coefficient values. Other PU algorithms comprise Transform-Domain PU adaptive filtering [2] that increases the convergence speed of PU-NLMS and data-selective PU adaptive filtering that decreases the computational complexity of PU-NLMS [6]. Some applications of PU adaptive filtering include Active Noise Control [7], [8] and channel equalization [9]. Some other applications in the literature comprise hands-free echo cancellation [10] and electroencephalographic signal analysis [11]. Complex versions of PU algorithms have been developed for channel identification [12].

In this paper, thresholds are proposed for PU algorithms updating a varying number of coefficients for channel identification and echo cancellation applications. In [4], [5], a fixed number of coefficients is updated. In the algorithms proposed here, the updated coefficients are defined in comparison to a gradient threshold and a coefficient threshold. The proposed algorithms are alternatives to the PU-NLMS M -max with a variable M . The update is randomized in one of the proposed algorithms, when the updating criteria are not met. In this paper, the complex versions of the proposed algorithms are

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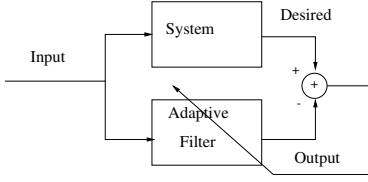


Fig. 1. Adaptive filter for system identification [1].

evaluated in the wireless channel scenario.

Section II shows the PU adaptive filters in the system identification configuration. Section III presents the proposed algorithms. Sections IV and V analyze computational complexity and convergence of the proposed algorithms, respectively. In Section VI, the algorithms are simulated, and the sensitivity is analyzed. Finally, Section VII concludes this work.

Regarding mathematical notation, boldface fonts are employed for vectors and matrices, where matrices are represented by capital letters. The complex conjugate is denoted by $*$, and the conjugate transpose is given by ^{H} . Moreover, $\mathbb{R}$ represents the field of real numbers, while $\mathbb{C}$ stands for the field of complex numbers.

II. PARTIAL UPDATE ADAPTIVE SYSTEM IDENTIFICATION

Consider an adaptive filter [1] with an input $x(k)$, desired signal $d(k)$, and coefficient vector $\mathbf{w}(k) = [w_0(k) \ w_1(k) \ \dots \ w_N(k)]^T$ in the system identification configuration, as shown in Fig. 1. The input to the adaptive filter is vectorially modeled as $\mathbf{x}(k) = [x(k) \ x(k-1) \ \dots \ x(k-N)]^T$. For the system identification configuration, the adaptive filter and the system to be identified share the same input. The output of the system to be identified is the desired signal $d(k)$ of the adaptive filter. The error signal is $e(k) = d(k) - y(k)$, where $y(k)$ is the output signal of the adaptive filter. For PU adaptive filtering, a criterion is defined to update part of $\mathbf{w}(k)$ at a given iteration [2] modeled by the diagonal selection matrix $\mathbf{C}(k)$, that is, the coefficient vector $\mathbf{w}_{up}(k) = \mathbf{C}(k)\mathbf{w}(k)$ is updated. The other coefficients are kept equal to the corresponding values in $\mathbf{w}(k-1)$.

For the PU-NLMS M-max algorithm, the i^{th} coefficient $w_i(k)$ is updated, whenever it corresponds

to one of the M highest inputs. Otherwise, it is not updated, that is, [4]

$$w_i(k+1) = \begin{cases} w_i(k) + \frac{\mu}{\mathbf{x}^T(k)\mathbf{x}(k)} e(k)x(k-i+1), & i \in \mathbb{A}, \\ w_i(k), & i \notin \mathbb{A}, \end{cases} \quad (1)$$

where the set $\mathbb{A}$ contains the indices i corresponding to the largest gradient elements at iteration k .

SPU measures the variance of the coefficients $w_i(k)$ at the k^{th} iteration [5]. The coefficients whose amplitude are greater than 3 standard deviations are updated. On the other hand, the coefficients within 3 standard deviations have a probability of update f .

The PU-NLMS M-max update is categorized as the Sequential-based PU algorithms, whereas SPU is classified as a Stochastic-based PU algorithms, according to [6]. The proposed algorithms have characteristics similar to both Sequential-based as well as Stochastic-based algorithms.

PU-NLMS M-max requires internally a sorting algorithm for ordering the coefficients in each iterations. Moreover, it uses a fixed number M of updated coefficients, which is not the best choice for time-varying environments. On the other hand, SPU uses a stochastic approach for selecting its coefficients to be updated. This paper aims to merge both approaches into a novel algorithm. The proposed algorithms are then presented in the next section.

III. PROPOSED PARTIAL-UPDATE DOUBLE THRESHOLD ALGORITHMS

The PU NLMS M-max algorithm [4] updates only the elements with the largest gradient entries. Although the value of M has an influence on its convergence [13], M is often chosen by considering the compromise between the convergence and the computational complexity of the algorithm. In Set-Membership filtering [14], a threshold is defined for selecting the element to be updated. This paper proposes a flexible selection for PU algorithms by using two thresholds in the selection of the coefficients, which are denominated as the Partial Update NLMS Double Threshold (PU-NLMS-DT) algorithms. Consider the cost function of the PU-NLMS-DT algorithms [15]:

$$\begin{aligned} \min_{\mathbf{w}(k+1)} \quad & \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2, \\ \text{s.t.} \quad & \mathbf{w}^T(k+1)\mathbf{C}^T(k)\mathbf{x}(k) = d(k). \end{aligned} \quad (2)$$

The constrained problem in Eq. (2) is equivalent to the Lagrangian expression:

$$\min_{\mathbf{w}(k+1), \lambda} \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 + \lambda^T (\mathbf{w}^T(k+1) \mathbf{C}^T(k) \mathbf{x}(k) - d(k)), \quad (3)$$

which leads to the following coefficients' updates:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\gamma + \mathbf{x}^T(k) \mathbf{C}(k) \mathbf{x}(k)} e(k) \mathbf{C}(k) \mathbf{x}(k). \quad (4)$$

Consider $\mathbf{x}_{up}(k) = \mathbf{C}(k) \mathbf{x}(k)$, that is, $\mathbf{x}_{up}(k)$ is identical to $\mathbf{x}(k)$ for the entries corresponding to the coefficients to be updated, and zero for the other elements. Therefore, $\mathbf{x}^T(k) \mathbf{x}(k) = \|\mathbf{x}(k)\|^2$ and $\mathbf{x}_{up}^T(k) \mathbf{x}(k) = \mathbf{x}^T(k) \mathbf{C}(k) \mathbf{x}(k) < \|\mathbf{x}(k)\|^2$. In this situation, $\mathbf{x}_{up}^T(k) \mathbf{x}(k)$ results in a smaller value in the denominator, which leads to an algorithm prone to be unstable due to the larger variability in the denominator value. After simulations for algorithms with thresholds, the presence of $\mathbf{C}(k)$ in the denominator is verified to lead to the update of fewer coefficients and a poor convergence.

A different update is used in the PU-NLMS M-max algorithm [4], with a modified normalization of the step size. It leads to the coefficient update [15] of the proposed algorithms:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\gamma + \mathbf{x}^T(k) \mathbf{x}(k)} e(k) \mathbf{C}(k) \mathbf{x}(k), \quad (5)$$

where $\mathbf{C}(k) \in \mathbb{R}^{N \times N}$ is a diagonal matrix. Consider $C_{ii}(k)$ as the i^{th} diagonal element of $\mathbf{C}(k)$:

$$C_{ii}(k) = \begin{cases} 1, & \text{if } |g_i(k)| \geq \delta_1 \text{ or } |w_i(k)| \geq \delta_2, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

where the threshold δ_1 is compared to the gradient estimate vector's elements, threshold δ_2 is related to the absolute value of the i^{th} coefficient element, and $g_i(k)$ is the i^{th} element of the gradient estimate $\mathbf{g}(k)$.

For time-varying system identification, $\mathbf{C}(k)$ of Eq. (6) may lead to a delay in the convergence. Then, PU-NLMS-DT with Selective Partial Update (PU-NLMS-DT-SPU) is proposed where there is a probability of update, if the original threshold criteria are not met, which incorporates some stochastic mechanisms from the SPU algorithm [5]. In this case, $C_{ii}(k) = 1$ with probability p and 0 with probability $1 - p$.

The complex version for the coefficient update of the proposed algorithm is [15], [16]:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\gamma + \mathbf{x}^H(k) \mathbf{x}(k)} e^*(k) \mathbf{C}(k) \mathbf{x}(k), \quad (7)$$

where $\mathbf{x}(k), \mathbf{w}(k) \in \mathbb{C}^N$ and $\mathbf{C}(k) \in \mathbb{C}^{N \times N}$.

Algorithm 1 Proposed PU-NLMS-DT and PU-NLMS-DT-SPU algorithms.

Consider $\mathbf{g}(k) = e(k) \mathbf{x}(k)$.

if $|g_i(k)| \geq \delta_1$ or $|w_i(k)| \geq \delta_2$, **then**

$C_{ii}(k) = 1$;

else

PU-NLMS-DT:

$C_{ii}(k) = 0$;

PU-NLMS-DT-SPU:

$r = \text{RAND-UNIFORM}()$;

if $r > p$, **then**

$C_{ii}(k) = 1$;

else

$C_{ii}(k) = 0$;

end if

end if

Real: $\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\gamma + \mathbf{x}^T(k) \mathbf{x}(k)} e(k) \mathbf{C}(k) \mathbf{x}(k)$

Complex: $\mathbf{w}(k+1) = \mathbf{w}(k) + \frac{\mu}{\gamma + \mathbf{x}^H(k) \mathbf{x}(k)} e^*(k) \mathbf{C}(k) \mathbf{x}(k)$

The proposed PU-NLMS-DT and PU-NLMS-DT-SPU are summarized in Algorithm 1. The use of a threshold for the value of instantaneous gradient estimate and another one based on the coefficient magnitude controls the convergence of both proposed algorithms. The updates when the criteria are not met address a high variability of the system response to be identified for the PU-NLMS-DT-SPU. The proposed algorithms are versions of PU-NLMS M-max where the number of updating coefficients is time-variant, whereas PU-NLMS M-max keeps constant M , that is, the number of updated coefficient elements.

In the proposed algorithms, the use of thresholds δ_1 and δ_2 have the following goals:

- The threshold δ_1 allows the coefficient update whenever the gradient estimate magnitude is large enough;
- The threshold δ_2 allows the coefficient update when its magnitude is large enough;
- For the proposed PU-NLMS-DT-SPU, it is allowed the coefficients to be sometimes updated when the previous two criteria are not met.

Since the proposed algorithms are PU adaptive filters, we aim to present a method to efficiently choose the coefficients to be updated. Therefore,

it is interesting to perform a comparison of the computational complexity proposed algorithm to other PU adaptive filters.

IV. COMPUTATIONAL COMPLEXITY

In this section, computational complexity of the proposed algorithm, PU-NLMS M-Max and SPU are compared in terms of flops (floating point operations) [17]. The proposed algorithms may be considered as a PU-NLMS M-Max with a variable number of coefficients to be updated. To express this equivalence, the number of updated coefficients at a given iteration is denominated as M for PU-NLMS M-Max, following the original denomination, and $P(k)$ for the proposed algorithms. The NLMS-based PU algorithms use a common formulation for updates, requiring $2N - P(k) + 1$ scalar multiplications or divisions and $2N - P(k) + 1$ additions, where $P(k)$ coefficients are updated in the k^{th} iteration. PU-NLMS M-max requires a sort algorithm for their coefficients, which increases its complexity by $N \log N$ flops. For SPU, the computational complexity depends on the parameter f , which represents the probability of update for each coefficient. On the other hand, L is a parameter from SPU [5] that represents the inverse of the ratio of coefficients to be updated, that is, $L = N/M$.

For the proposed algorithms, a gradient estimation is necessary, where $e(k)\mathbf{x}(k)$ is used as the instantaneous estimate. The multiplication $e(k)\mathbf{x}(k)$ is already used in the update, then a small overhead is introduced in this update. Instead of using the M largest coefficients, the proposed algorithms compare the coefficients to a threshold. Comparison of a vector with a fixed scalar threshold leads to the equivalent of the complexity of just one addition [17]. The computational complexity of the proposed algorithms compared to PU-NLMS M-max, NLMS and SPU is shown in Tab. I.

Proposed algorithms present a lower computational complexity than NLMS, PU-NLMS M-Max and SPU. PU-NLMS-DT-SPU has an extra step consisting of a random number generation for each non-updated coefficient. One digit of the inner hardware clock platform can generate this random number, since the requirements on the randomness of this number are pretty loose. Then, $1 - P(k)$ extra flops may be added for the k^{th} iteration for the proposed PU-NLMS-DT-SPU. The reduction

TABLE I
NUMBER OF FLOPS OF THE PROPOSED ALGORITHMS COMPARED TO OTHER PU ALGORITHMS.

Operation	NLMS	NLMS SPU	PU M-max	Proposed Algorithms
Multipl.	$2N+1$	$2N+M+fML$	$2N-M+1$	$2N-P(k)+1$
Addition	$2N+1$	$2N+M+fML$	$2N-M+1$	$2N-P(k)+1$
Compar.	–	fN	–	1
Sort	–	–	$N \log N$	–

in computational complexity may interfere in the convergence of the algorithms. The next section provides a condition for the proposed algorithm to converge.

V. CONVERGENCE ANALYSIS

The convergence of the NLMS is guaranteed for $\mu \leq 2$ [1]. All the PU algorithms update part of the coefficients, which modifies the range of convergence of the step size μ [13]. Our MSE convergence analysis is deduced by adapting [13] to the proposed algorithms. The quantity $P(k)$ of updated taps varies over time, but the final MSE is proven to be the same regardless of the values of $P(k)$, if $\mathbf{C}(k) \neq \mathbf{0}, \forall k$ [13]. Input signal $x(k)$ is assumed to be white with a Gaussian distribution. The indices to be updated in $\mathbf{C}(k)$ is assumed to be independent across the iterations.

Consider that $\Delta \mathbf{w}(k) = \mathbf{w}(k+1) - \mathbf{w}(k)$, then

$$e(k+1) = e(k) + \sum_{n=1}^N \frac{\partial e(k)}{\partial w_n(k)} \Delta w_n(k), \quad (8)$$

where $\Delta w_n(k)$ is the n^{th} element of $\Delta \mathbf{w}(k)$, and $e(k+1) = d(k+1) - y(k+1)$. For the proposed PU-NLMS-DT,

$$\Delta w_n(k) = \frac{\mu}{\gamma + \mathbf{x}^T(k)\mathbf{x}(k)} e(k)x(k-n+1), \quad (9)$$

for $n \in P(k)$, or $\Delta w_n(k) = 0, n \notin P(k)$. By substituting the value of $\Delta w_n(k)$ in the error expression:

$$e(k+1) = e(k) + \frac{\mu}{\gamma + \mathbf{x}^T(k)\mathbf{x}(k)} \sum_{n=1}^N x(k-n+1), \quad (10)$$

and then squaring both sides of (10),

$$e^2(k+1) = e^2(k) + \frac{\mu^2}{(\gamma + \mathbf{x}^T(k)\mathbf{x}(k))^2} \sum_{n=1}^N x^2(k-n+1). \quad (11)$$

The number of updated coefficients $P(k)$ at the k^{th} iteration is time varying. None of the elements may be updated for $P(k) = 0$, where $\mathbf{w}(k+1) = \mathbf{w}(k)$. This situation is trivial, without requiring further analysis for this iteration. On the other hand, if all the coefficients are updated at a given iteration, the proposed algorithms correspond to the NLMS update.

In this paper, we aim to reach an upper limit for μ . This upper limit tends to be tighter for extreme scenarios of difficult convergence. The most extreme situation for convergence is the update is the iteration k where only one coefficient is updated. This is the situation analyzed here, considering the worst case scenario. For the proposed algorithm to converge, the choice of μ should be adequate for all possible values of $P(k)$.

Consider the Wiener solution $\mathbf{w}_{opt}$ [1], and the auxiliary coefficient error vector $\mathbf{v}(k) = \mathbf{w}(k) - \mathbf{w}_{opt}$ [13]. It is shown in [4], [13] that, when one coefficient is updated,

$$\begin{aligned} v_n(k+1) &= (1 - \frac{\mu}{q(k)} x^2(k-n+1))v_n(k) \\ &- \frac{\mu}{q(k)} \sum_{l \neq k} x(k-n+1)x(k-l+1)v_l(k) \\ &+ \frac{\mu}{q(k)} x(k-n+1)e_{opt}(k), \end{aligned} \quad (12)$$

for the updating coefficient, and $v_n(k+1) = v_n(k)$, otherwise, where $e_{opt}(k) = d(k) - \mathbf{w}_{opt}^T(k)\mathbf{x}(k)$ and $q(k) = \gamma + \mathbf{x}^T(k)\mathbf{x}(k)$.

Consider that, at the k^{th} iteration, the u^{th} coefficient is updated. Then [4], [13],

$$\begin{aligned} E[v_u^2(k+1)] &= (1 - 2\mu_N\sigma_x^2 + \mu_N^2\eta)E[v_u^2(k)] \\ &+ \mu_N^2\sigma_x^4 \sum_{l=1, l \neq u} E[v_l^2(k)] + \mu_N^2\sigma_x^2\epsilon_m, \end{aligned} \quad (13)$$

where $E[\cdot]$ denotes statistical expectancy, $\eta = E[x^4(k)]$, $\epsilon_m = E[e_{opt}^2(k)]$, and $\mu_N = \frac{\mu}{N\sigma_x^2}$. If $l \neq u$, $E[v_l^2(k+1)] = E[v_l^2(k)] = D(k)$, therefore,

$$\begin{aligned} D(k+1) &= \left(1 - 2\frac{\mu_N}{N}\sigma_x^2 + \frac{\mu_N^2}{N}(\eta + (N-1)\sigma_x^4)\right) D(k) \\ &+ \frac{\mu_N^2}{N}\sigma_x^2\epsilon_m. \end{aligned} \quad (14)$$

The convergence is guaranteed for the following range

$$0 < \mu < \frac{2N\sigma_x^4}{\eta + (N-1)\sigma_x^4}. \quad (15)$$

The convergence is still possible for μ outside of the range defined in Eq. (15), since the deduction is performed for the worst case scenario, when one coefficient is updated.

Consider the mathematical derivation in [18] specifically for the worst case scenario, that is, $P(k) = 1$, then $0 < \mu_2 < \frac{2}{N}$. Since our proposal does not use blocks of coefficients, the deduction in [18] was then adapted. The update in [18] is normalized by $\mathbf{x}^T(k)\mathbf{C}(k)\mathbf{x}(k)$, unlike Eq. (5).

From Eq. (11), a deduction may be performed as in [19], for a fixed $M > 1$.

As shown in [19],

$$0 < \mu < \frac{\|\mathbf{x}(k)\|^2}{\|\bar{\mathbf{x}}(k)\|^2}, \quad (16)$$

where $\bar{\mathbf{x}}(k) = \mathbf{C}(k)\mathbf{x}(k)$. For $M = 1$, then $\|\bar{\mathbf{x}}(k)\|^2 = D(k)$.

Since $\|\bar{\mathbf{x}}(k)\|^2$ measures the power of some terms of $\mathbf{x}(k)$, then $\frac{\|\mathbf{x}(k)\|^2}{\|\bar{\mathbf{x}}(k)\|^2} > 1$. The range of μ in a normalized update is duplicated around 1 [1]. Then, the PU-NLMS Mmax algorithm is guaranteed to converge for $0 < \mu < 1$, which is equivalent to the original NLMS constraint. Since the proposed algorithms may be viewed as a variable M version of the PU-NLMS Mmax algorithm, then a similar reasoning may be used.

In the next section, the proposed algorithms are simulated, as well as the benchmark algorithms.

VI. RESULTS

Computer simulations have been performed in Matlab 2017a for evaluating the performance of the proposed algorithms. Firstly, the thresholds of the proposed algorithm are properly set. Then, the MSE of the proposed algorithm is compared to other PU algorithms.

A. Setting the Thresholds

In this subsection, some simulations provide a rough evaluation of the sensitivity of the proposed algorithm to the values of the thresholds. The sensitivity to a parameter δ is defined as $S = \frac{\partial e^2(k)}{\partial \delta}$ [24]. Since it is a quite difficult variable to be evaluated, the estimation of the sensitivity should be performed, as in the analysis developed here. In this subsection, the main objective is to study the influence of the thresholds over the MSE. Then,

the use of a rougher estimate of the sensitivity is enough to obtain a proper conclusion about how the threshold values influence the MSE.

A sensitivity estimate may be defined, such that $\hat{S}_{\delta x} = \lim_{\Delta\delta_x \rightarrow 0} \frac{\Delta e^2(k)}{\Delta\delta_x}$, where the overbar denotes time averaging. In this subsection, a low-complexity rougher estimate is used $S_{\delta x}(\delta_{x1}, \delta_{x2}, k) = \frac{e_{\delta x2}^2(k) - e_{\delta x1}^2}{\delta_{x2} - \delta_{x1}}$, as depicted in Fig. 2 for the sensitivity to δ_1 in the basic scenario. The results for sensitivity to δ_2 are very similar, then they were omitted. The results show a very low sensitivity of the proposed PU-NLMS-DT to parameters δ_1 and δ_2 in the basic scenario.

B. Sensitivity: Basic Scenario

The basic scenario is the identification of a Digital Subscriber Line (DSL) cable channel with a time-invariant impulse response as in [20], that is, a base-band 120 MHz cable with 50-m length. A low-pass frequency response similar to [20] is used, with its corresponding impulse response decimated in time for a 16-tap digital channel. A Bernoulli-Gaussian impulsive noise is added to the signal [21]. Figs. 2 and 3 as well as Tabs. II and III show the main results for sensitivity. There is a low sensitivity for both thresholds for this scenario.

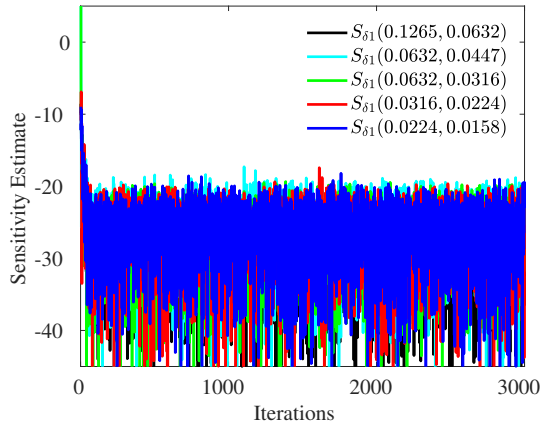


Fig. 2. Sensitivity of the proposed PU-NLMS-DT for values of δ_1 in the basic scenario.

C. Sensitivity: Echo Cancellation

For acoustic echo cancellation, $x(k)$ corresponds to a short sentence read by a male speaker with

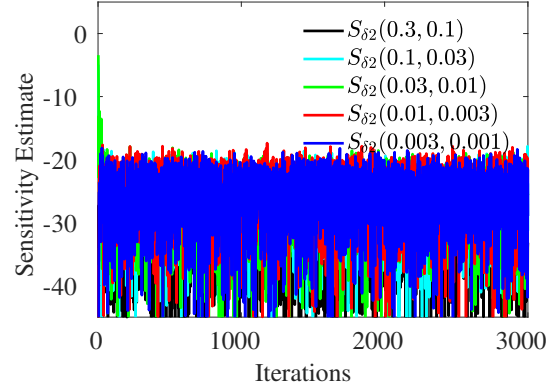


Fig. 3. Sensitivity of the proposed PU-NLMS-DT for values of δ_2 in the basic scenario.

TABLE II
SENSITIVITY AVERAGE FOR VALUES OF δ_1 IN THE BASIC SCENARIO.

$\delta_1(x, y)$	$\bar{S}_{yx}$
(0.1265, 0.0632)	-27.41 dB
(0.0632, 0.0447)	-22.71 dB
(0.0447, 0.0316)	-22.70 dB
(0.0316, 0.0224)	-25.33 dB
(0.0224, 0.0158)	-25.01 dB

some silent parts and sampled at 44.1 kHz. The system to be identified presents a time-invariant response. The echo path used is a simplified version of the echo paths from [22], which represents a 4-kHz baseband transmission channel. The first coefficient is set as 1 and the eighth coefficient is -0.5 of a 16-tap echo path. Figs. 4 and 5 as well as Tabs. IV and V show the sensitivity estimates of PU-NLMS-DT for the audio scenario to δ_1 and δ_2 , respectively. The sensitivity estimates are quite high to δ_1 and low to δ_2 .

D. Sensitivity: Channel Identification

The next set of simulations comprises a wireless channel identification, whose requirements of tracking of the algorithms are quite demanding.

TABLE III
SENSITIVITY AVERAGE FOR VALUES OF δ_2 IN THE BASIC SCENARIO.

$\delta_2(x, y)$	$\bar{S}_{yx}$
(0.3, 0.1)	-32.50 dB
(0.1, 0.03)	-7.42 dB
(0.03, 0.01)	-5.93 dB
(0.01, 0.003)	-23.86 dB
(0.003, 0.001)	-24.61 dB

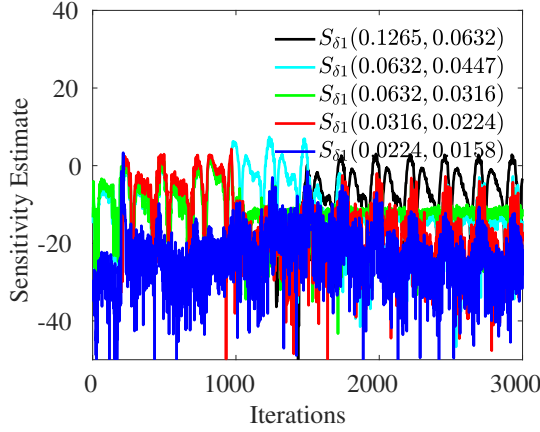


Fig. 4. Sensitivity of the proposed PU-NLMS-DT for values of δ_1 for echo cancellation.

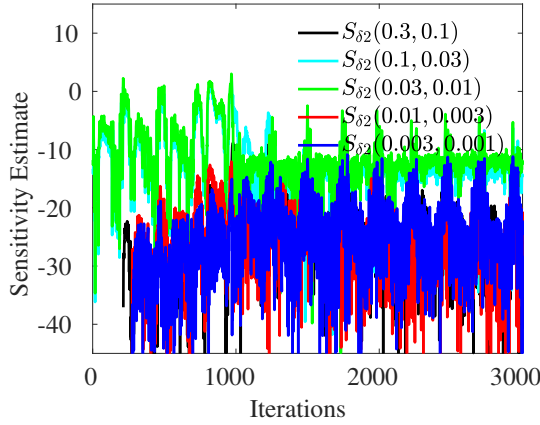


Fig. 5. Sensitivity of the proposed PU-NLMS-DT for values of δ_2 for echo cancellation.

The sparse impulse response changes once every 10 iterations, where 2 out of the 16 taps are chosen to be non-zero, while the other 14 are zeroed out. The transmission is assumed to be in bandpass at 815 MHz. Since the system is simulated in baseband, the complex versions of the algorithms are used. Fig. 6 and Tab. VI show the sensitivity estimates of PU-NLMS-DT to δ_1 for channel identification. The

TABLE IV
SENSITIVITY AVERAGE FOR VALUES OF δ_1 FOR ECHO CANCELLATION.

$\delta_1(x, y)$	$\bar{S}_{yx}$
(0.1265, 0.0632)	-6.02 dB
(0.0632, 0.0447)	-4.25 dB
(0.0447, 0.0316)	-8.23 dB
(0.0316, 0.0224)	-7.44 dB
(0.0224, 0.0158)	-16.71 dB

TABLE V
SENSITIVITY AVERAGE FOR VALUES OF δ_2 FOR ECHO CANCELLATION.

$\delta_2(x, y)$	$\bar{S}_{yx}$
(0.3, 0.1)	-19.29 dB
(0.1, 0.03)	-9.12 dB
(0.03, 0.01)	-8.23 dB
(0.01, 0.003)	-21.52 dB
(0.003, 0.001)	-20.23 dB

sensitivity to δ_2 is not depicted, since the results are similar to the ones shown in Fig. 6, i.e., low sensitivity with some oscillations to the variability of the channel.

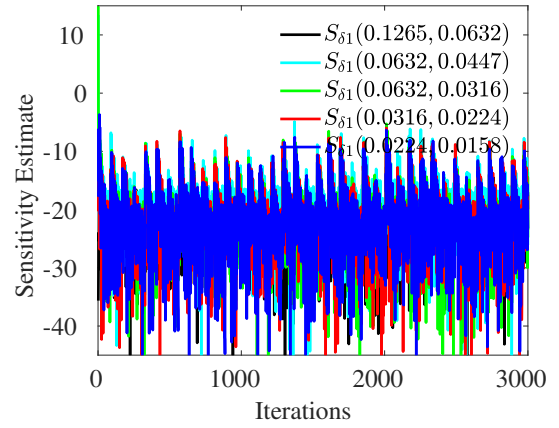


Fig. 6. Sensitivity of the proposed PU-NLMS-DT for values of δ_1 for channel identification.

In summary, the proposed algorithms present generally low sensitivity to the values of the thresholds in relation to the squared error. Then, the proposed algorithms may be tuned such that they present low computational complexity and low MSE values. The values for the thresholds used in the MSE simulations are chosen within the range $[10^{-3}, 10^{-2}]$, that is, in the range simulated in this sensitivity analysis. For the simulations in this section, it is not important the actual values of the sensitivity estimation curve. In fact, the focus of this analysis is the variability of the curves when a threshold is varied.

E. MSE simulations: Basic Scenario

Modern systems feature high variability due to mobility and high rates, leading to sparse channel responses. The proposed PU-NLMS-DT and PU-NLMS-DT-SPU algorithms are tested in communication scenarios to identify channel responses. Four

TABLE VI
SENSITIVITY AVERAGE FOR VALUES OF δ_1 FOR CHANNEL IDENTIFICATION.

$\delta_1(x, y)$	$\bar{S}_{yx}$
(0.1265, 0.0632)	-19.92 dB
(0.0632, 0.0447)	-9.85 dB
(0.0447, 0.0316)	-8.82 dB
(0.0316, 0.0224)	-18.89 dB
(0.0224, 0.0158)	-19.02 dB

scenarios are used in the Matlab 2017a simulations: a basic scenario, an acoustic echo cancellation scenario, and two wireless channels with a time-varying impulse response presenting a high variation.

The algorithms in simulation are NLMS, SPNLMS [3], PU-NLMS Mmax [4] and SPU [5], whose MSE are compared to the proposed PU-NLMS-DT and PU-NLMS-DT-SPU. Simulations are performed for 4,000 input symbols and 600 ensemble runs, i.e., the results correspond to the average of the MSE over 600 Monte Carlo runs. The number of Monte Carlo runs was chosen so that the results are constant, if this set of simulations is repeated. The thresholds are chosen to keep the same update rates for the algorithms. For the wireless scenarios, the complex versions of the algorithms have been used, since the actual transmission is performed in passband. The transmitted constellation used is a normalized 256 Pulse Amplitude Modulation. A measurement noise with -30 dBm is added to the transmission in the channel.

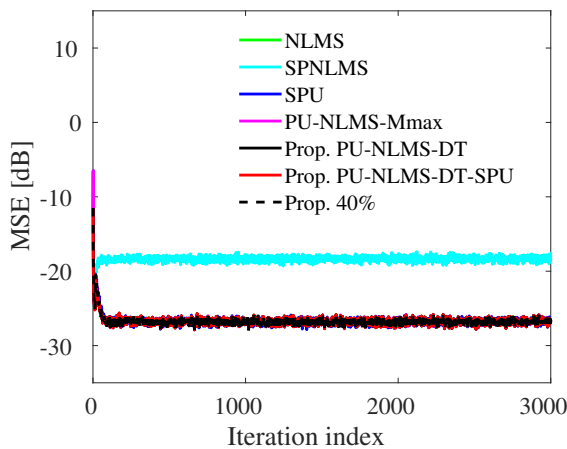


Fig. 7. DSL cable identification when the update rates of all simulated algorithms are set to 81%, except for NLMS and PU-NLMS-DT-SPU with 40% rate.

The basic scenario is now revisited for the MSE evaluation. A base-band 120 MHz cable with 50-

m length with a low-pass frequency response is identified after decimated in time for a 16-tap digital channel. An impulsive noise is added to the signal. A fixed rate of iterations is enforced for every algorithm by setting the parameters properly. Fig. 7 shows the results, when all algorithms are set to an update rate of 81%, except for the NLMS algorithm. Most proposed and benchmark algorithms present similar MSE, and SPNLMS presents the highest MSE. Moreover, the proposed PU-NLMS-DT-SPU is also shown for 40% of update to show the low sensitivity of the proposed algorithms previously presented in this section.

In all of the simulations, the color of the curves is maintained. MSE of the NLMS is depicted in green, SPNLMS in cyan, SPU in blue and PU-NLMS-Mmax in magenta. On the other hand, the MSE of the proposed PU-NLMS-DT is represented in solid black and the proposed PU-NLMS-DT-SPU in red. Moreover, the proposed algorithms are also simulated in a reduced update mode, depicted in dashed black.

F. MSE simulations: Acoustic Echo Cancellation

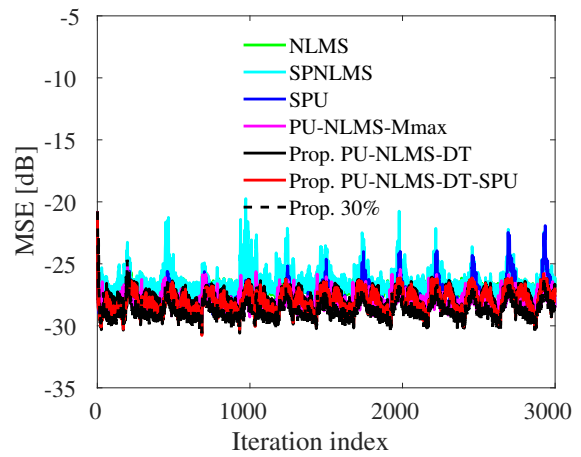


Fig. 8. Echo cancellation simulations where all the algorithms have a fixed update rate of 50%, except for NLMS and PU-NLMS-DT-SPU with 40% rate.

The acoustic echo cancellation scenario is once more used, where the response corresponds to a short sentence read by a male speaker with some silent parts and digitized at a sample rate of 44.1 kHz. The system to be identified presents a sparse impulse response, with the first coefficient set as 1 and the eighth coefficient is -0.5 of a 16-tap

response. Fig. 8 shows the simulations for fixed 50% update rate for all the algorithms by properly setting the parameters, except for the NLMS algorithm that does not provide this compromise. The curve for the proposed PU-NLMS-DT-SPU is also shown for 30% of updates, which confirms the low sensitivity of the proposed algorithms. The proposed algorithms and the PU-NLMS M-max present the best MSE performance and fastest convergence for echo cancellation.

G. MSE simulations: Wireless Channel Identification

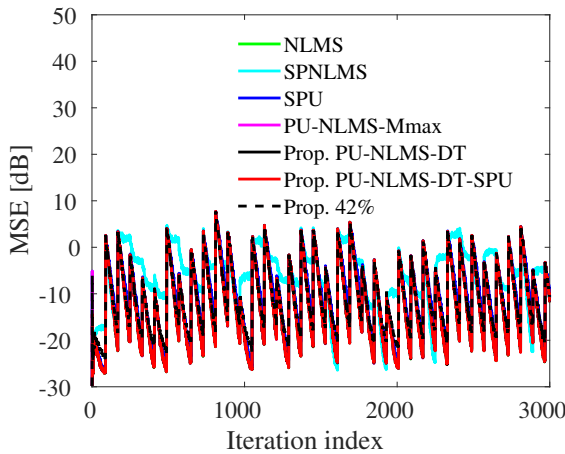


Fig. 9. Wireless channel identification for the complex versions of the algorithms with 81% of update rate, except for NLMS and PU-NLMS-DT-SPU with 40% rate.

The wireless channel identification scenario is used for MSE evaluation, where the impulse response changes once every 10 iterations, and 2 out of the 16 taps are chosen to be non-zero, while the other 14 are zeroed out. The complex version of the algorithms are simulated. In Fig. 9, the proposed PU-NLMS-DT-SPU present the best MSE performance regarding MSE, while 81% of the coefficients along the iterations are kept fixed.

The Extended Pedestrian A (EPA) channel for LTE is used for time-varying channel identification [23], with a truncated 52-tap impulse response in bandpass at 1800 MHz, for a speed of 2 m/s. The complex versions of the algorithms are used in this simulation, since bandbase representation of signals is used. Fig. 10 shows the results for the algorithms in the EPA channel, when the parameters of all the algorithms are set to be kept at 68% coefficient

updates, except for the NLMS. In this scenario, the simulations confirm the low sensitivity regarding the threshold values of the proposed algorithm by displaying the curve for PU-NLMS-DT-SPU with 42% of updates. The PU-NLMS M-max and proposed PU-NLMS-DT and PU-NLMS-DT-SPU present the lowest MSE.

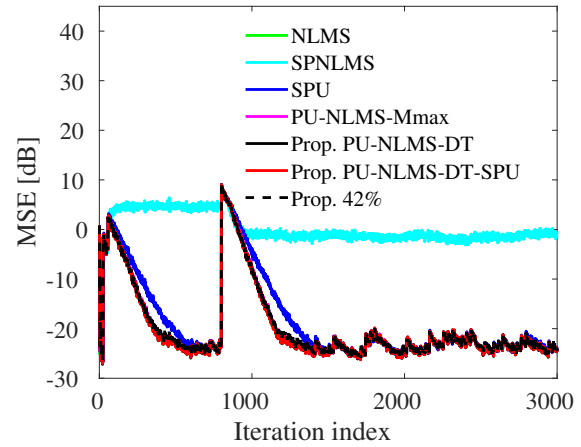


Fig. 10. MSE for the EPA-LTE 52-tap channel identification for the complex versions of the algorithms by keeping 68% of update rate, except for NLMS. The proposed PU-NLMS-DT-SPU is set to 40% of update rate.

In the simulated scenarios, the step size is always smaller than the upper bound deduced in Section V, that is, within the convergence range that is deduced in Eq. (15). Since the bound is deduced in the worst case scenario, one obtains 0.125 as a pessimistic upper limit for μ in most of the simulated scenarios.

The proposed algorithms present similar MSE performance to the benchmark algorithms. The proposed algorithms exhibit a faster convergence, especially for the acoustic echo cancellation and LTE channel identification scenarios, i.e., when the input is non-stationary or the channel response is time-varying. The final excess MSE for all the compared algorithms are similar, which is predicted theoretically [13], while the computational complexity of the proposed algorithms is generally lower than most of the other algorithms under the same conditions. The exception is PU-NLMS M-max which requires a sorting algorithm for selecting the largest inputs in each iteration, as shown in Section IV. In the simulated scenarios, the step size is always smaller than the upper bound deduced in Eq. (15). The next section concludes this work.

VII. CONCLUSION

This paper proposes the use of a time-varying number of updating coefficients selected by using two thresholds for defining the coefficients updated at each iteration for channel identification and echo cancellation.

Simulations show that the proposed algorithms converge faster than SPU for acoustic echo cancellation, which uses a non-stationary input, and LTE standardized channel identification, which presents high variability in time. For the time-varying wireless impulse response, the convergence of the proposed algorithms is slightly faster. Moreover, the proposed algorithms present lower computational complexity than PU-NLMS M-Max while maintaining a similar convergence.

A convergence analysis of the proposed algorithms' step sizes and a sensitivity analysis of the thresholds based on simulations are provided. The proposed algorithms present different sensitivity regarding MSE and update rates in the simulated scenarios. The sensitivity simulations generally show that the proposed algorithms allow the MSE to be maintained while the update rate is significantly reduced. A low-complexity proposed algorithm is reached without compromising the MSE. The proposed PU-NLMS-DT algorithm may be viewed as a version of the PU-NLMS M-max algorithm with a time-varying M . As a future work, performance tests will be performed on active noise control in a hands-free vehicular scenario [25], [26].

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