Ground-Reflection Analytical Approach for Horizontally Variable Refractive Index Condition

Diego Parada Rozo, Dinael Guevara, Alexandre Rocha, Diego Tami and Cássio G. Rego

***Abstract*—An innovative formulation for the calculation and analysis of ground-reflection of ray paths under condition of horizontally variable refractivity is introduced in this letter. The deduced formulation is incorporated within a modified Ray Tracing (RT) technique that considers atmospheric refractivity effect in radiopropagation modeling. Canonical tests were carried out, where the proposed analytical solution was applied to predict coverage in a long-distance scenario that assumed different configurations of horizontally stratified refractivity changes along the investigated environment. The results achieved were confronted with the numerical solution of the Split-Step Fourier Parabolic Equation (SSPE) at 10 GHz. It was possible to observe that the results obtained from a modified RT with horizontally variable refractivity and the SSPE method have a similar behavior when estimating the received power. These innovations contribute to the development of algorithms for modeling and analyzing wireless networks in more realistic environments.**

***Index Terms*—Ray tracing, multipath analytical approach, horizontally variable refractivity, ground-reflection.**

I. Introduction

RECENT progress in wireless technologies presents several challenges for radio-channel characterization, especially due to operation at high frequencies and large bandwidths [1]. Such radio-channel characterization becomes more complex when non-homogeneous atmospheric conditions are imposed to the radiowave propagation. Consequently, it is very important to develop fast and accurate propagation models that allow the inclusion of the majority of these atmospheric effects. Furthermore, these models must consider realistic environmental factors to ensure accurate and consistent results [2].

Many studies of two-dimensional (2-D) environment to analyze radio propagation links are reported in the literature. In most of them, the numerical solutions using the Parabolic Equation (PE) model and Ray Tracing (RT) algorithms are considered the most useful effective ones [3], [4]. Both approaches are fast and reliable, enabling the propagation model with non-homogeneous environmental conditions, such as atmospheric refraction [5].

The RT is a well-established radio-channel characterization technique. It is based on the asymptotic methods of Geometrical Optics (GO) and Uniform Theory of Diffraction (UTD) [6]. Recently, Parada *et al.* [7] highlighted the importance of the RT technique to analyze tropospheric propagation and their advantages over numerical methods. The primary relevance of the RT technique is its ability to separate propagation paths and predict multipath components. These components are modeled as rays launched from a transmitter, interacting with the environment until they reach the receiver points. This multipath propagation allows the modeling of reflections, diffractions, free space attenuation, and multiple combinations of these phenomena [8].

The non-uniform spatial distribution of atmospheric refractivity can produce multipath fading and interference, which significantly affect the performance of long-distance wireless links [9]. The earth's troposphere presents more variations in the vertical than in the horizontal direction. However, the troposphere horizontal stratification may occur when a ray traveling partly over open terrain, or a suburban area, and partly through an urban enviroment, can then be exposed to various refractivity gradients along the propagation paths [10]. To consider atmospheric refractivity effect in radiopropagation modeling an RT formulation was reported by Valtr and Pechac [5], [11]. Their formulation is based on an analytical definition of the constant refractivity gradient along the height. Another work by these authors detailed the RT approach for a horizontally variable refractivity gradient [10]. However, the applicability of these solutions is limited to radio propagation with line-of-sight between transmitter and receiver in scenarios where the terrain influence is not relevant.

The horizontally variable refractivity gradient proposed by Valtr and Pechac in [10] considered a direct ray path solution, disregarding other propagation mechanisms, such as reflection and diffraction. Therefore, this work proposes a novel formulation to obtain an analytical solution that include the incident and reflected ray paths for the horizontally stratified variation of the refractivity gradient over flat terrain. Subsequently, this formulation is implemented using an RT algorithm, called by the authors as modified RT with atmospheric refractivity effect [7], [8]. To validate the proposed analytical approach, different variations of horizontally stratified refractivities are

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implemented in a canonical scenario. The simulation results of the modified RT algorithm are compared with the Fourier Split-Step Parabolic Equation (SSPE) numerical solution.

This letter is organized as follows: Section II presents the derived formulation of a ground-reflection analytical approach, the validation and simulation results are shown in Section III, and the conclusions are presented in Section IV.

II. Formulation Description

The light ray differential equation for the optical waveguide is the basis of the formulation focused in this work to ray path description, as specified in [12]. In addition, high frequency harmonic field, non-homogeneous media and slight changes of the refractive index value compared to the wavelength, are conditions assumed [10].

A. Ray path formulation for horizontally variable refractivity

Fig. 1 depicts the path of a direct ray as a function of horizontally stratified refractivity gradient variations. In this situation, the location of the transmitter and a receiver point are known.

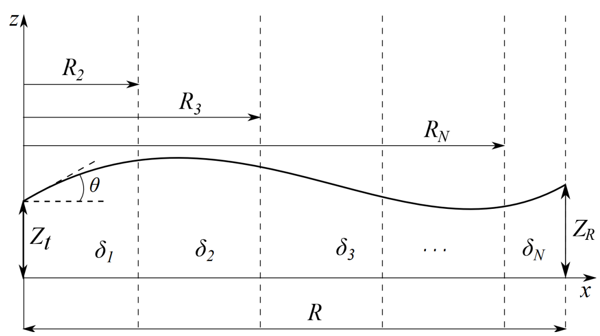

Fig. 1. Direct ray path for refractivity horizontal variation case.

Based on Fig. 1 with distance-dependent refractivity gradients, the following expression was presented in [10] to determine the ray path considering the mentioned refractivity conditions:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}x^2} = \delta_1 + \mathbf{1}(x - R_2)[-\delta_1 + \delta_2] + ... \\ + \mathbf{1}(x - R_n)[-\delta_{N-1} + \delta_N]. \tag{1}$$

As illustrated in Fig. 1, x is the distance, z is the height, Z_t is the transmitter height, Z_R is the height of a receiver point, R refers to the distance between the receiver point and the transmitter, δ_i represents the distance-dependent refractivity gradient, R_i are the distances related to those gradients, $R_1 = 0$ and $\mathbf{1}(p)$ corresponds to a unit step function that is given as follows:

$$\mathbf{1}(p) = \begin{cases} \mathbf{1}(p) = 0 & \text{for} \quad p \leq 0 \\ \mathbf{1}(p) = 1 & \text{for} \quad p > 0 \end{cases} \tag{2}$$

Equation (1) is solved in [10] by using Laplace transform and shifting theorem. In this way, the following equation is obtained as solution:

$$z = \frac{\delta_1 x^2}{2} + Kx + Z_t + \sum_{i=2}^{N} \mathbf{1}(x - R_i) \\ \times \left[\frac{\delta_i}{2}(x - R_i)^2 - \frac{\delta_{i-1}}{2}(x - R_i)^2 \right], \tag{3}$$

where K is the initial tangent, $K = \tan\theta$. Here, θ should be considered an approximate launch angle that assumes the condition of a rectilinear ray description, in order to simplify its calculation. Its importance lies in the fact that it is a starting parameter of the RT algorithm. The calculation of the launch angle was addressed in [7] for a case where the scenario is not horizontally stratified. However, this launch angle calculation was also used in the horizontal variation of refractivity conditions as can be seen in [10]. Thus, the following expression is used in our work to compute θ:

$$\theta = \arctan\left(\frac{\frac{\delta_1 R^2}{2} + Z_R - Z_t}{R} \right) \tag{4}$$

B. Ground-reflection formulation for horizontally variable refractivity

In a previous work, the current case of horizontal refractivity variation was used in scenarios that considered only direct-line ray paths [10]. Therefore, the authors of this letter propose the following approach to include ground-reflection analysis. Consequently, a novel formulation is derived and intended to be used in RT algorithms that include atmospheric refractivity effect.

The formulation is proposed in order to estimate the reflection point location. Fig. 2 exhibits the variables involved in the incident and reflected ray paths study. In this approach the location of a receiver point is known (R, Z_R) and X is the distance, measured horizontally, from the transmitter to the reflection point location. Initially, we isolate K in (3):

$$K = \frac{z - Z_t}{x} - \frac{\delta_1 x}{2} - \sum_{i=2}^{N} \mathbf{1}(x - R_i) \\ \times \left[\frac{(x - R_i)^2}{2x}(\delta_i - \delta_{i-1}) \right]. \tag{5}$$

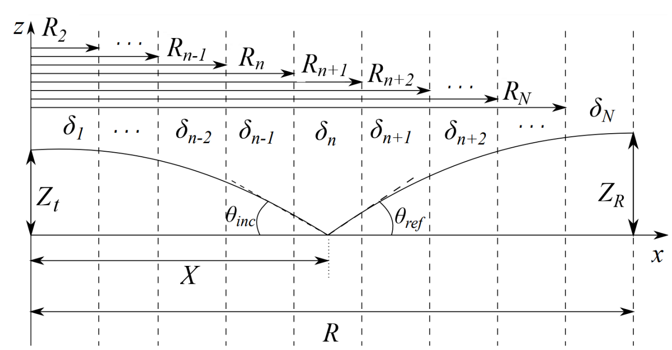

Fig. 2. Incident and reflected paths for refractivity horizontal variation case.

Based on Fig. 2 and the reciprocity theorem, we analyze the incident ray path. Here the following conditions were considered: $Z_t = 0$, $x = X$, $z = Z_t$ and $\delta_1 = \delta_n$, and substituting them in (5), an expression for K_{inc} is derived as follows:

$$K_{inc} = \frac{Z_t}{X} - \frac{\delta_n X}{2} - \sum_{i=2}^{n} \frac{R_i^2}{2X}(\delta_{i-1} - \delta_i). \tag{6}$$

In the same way, analyzing the reflected ray path and according to Fig. 2, the following conditions were considered: $Z_t = 0$, $x = R - X$, $z = Z_R$ and $\delta_1 = \delta_n$, which allows us to obtain an expression for K_{ref} as follows:

$$\begin{aligned} K_{ref} = \frac{Z_R}{R-X} - &\frac{\delta_n(R-X)}{2} \\ &- \sum_{i=n+1}^{N} \frac{(R-R_i)^2}{2(R-X)}(\delta_i - \delta_{i-1}). \end{aligned} \tag{7}$$

The following variable changes are then made:

$$A = \sum_{i=n+1}^{N} \tfrac{(R-R_i)^2}{2}(\delta_i - \delta_{i-1}), \tag{8}$$

$$B = \sum_{i=2}^{n} \tfrac{R_i^2}{2}(\delta_{i-1} - \delta_i), \tag{9}$$

$$C = \frac{\delta_n}{2}. \tag{10}$$

Consequently, (6) and (7) can be rewritten as:

$$K_{inc} = \frac{Z_t}{X} - CX - \frac{B}{X}, \tag{11}$$

$$K_{ref} = \frac{Z_R}{R-X} - C(R-X) - \frac{A}{(R-X)}. \tag{12}$$

From Snell's law: $K_{inc} = K_{ref}$, hence (11) and (12) are combined, leading to the following cubic equation:

$$\begin{aligned} (2C)X^3 - (3CR)X^2 + (A + B + CR^2 \\ - Z_t - Z_R)X + (RZ_t - BR) = 0. \end{aligned} \tag{13}$$

In the solution of (13), the following criteria must be met: $i) X = x_1$ is a real root and $ii)$ $R_n < X < R_{n+1}$. Therefore, the coordinate $(X, 0)$ corresponds to the location of the ground-reflection point. Subsequently once the ground-reflection point has been determined, it is possible to calculate the incident and reflected ray paths using (3).

III. Validation and Simulation Results

The novel formulation and proposed algorithm of a modified RT technique for horizontally variable refractivity are applied to different cases of refractivity change. To validate them, a comparison is made with the numerical SSPE algorithm, whose computational implementation has been previously evaluated [7], [8]. The choice of the SSPE method as a reference for validating the results is based on the fact that, although it is an approximate solution that considers a predominant direction of propagation, it offers accuracy, computational efficiency, and rapid convergence of results [3]. Furthermore, the use of SSPE is particularly suitable for the propagation problem addressed in this work, since numerical radiopropagation algorithms based on full-wave solutions (such as Finite-Difference Time-Domain, Method of Moments and Finite Element Method) would be unfeasible in terms of simulation time due to the marked inhomogeneity of the atmosphere considered. In this work, the SSPE algorithm was adjusted by the authors to consider the effects of horizontally variable refractivity.

The simulation conditions used coincide with the characterization of the scenarios investigated in [10]. Our purpose is to demonstrate the different response that can be identified when the ray description model is extended to include the ground-reflection formulation. The radiopropagation cases are analyzed for a perfectly conducting ground flat Earth, the simulations use a signal frequency at 10 GHz and horizontal polarization. In order to consider the effects of earth curvature in the solution, the refractive index gradient is substituted by a gradient of modified refractive index. Both the transmitter and the receiver points are located at a height of 30 meters along the investigated environment. To carry out the simulations, the source is represented using a Gaussian antenna beam pattern, a common choice in most 2D long-range radio propagation models. This approach allows for adjustments in beamwidth and beam tilt while providing an approximate representation of paraboloid dish antennas [13]. The received power results are plotted versus distance for all situations. The case tests presented in this Section maintain the simulation conditions carried out in [10] with the intention of highlighting our contribution by obtaining results that include the ground-reflection phenomenon within the analytical modeling for the refractivity condition mentioned.

Fig. 3 depicts the results with the standard refractivity gradient $\delta = -40$ N/km constant in the whole distance range. The solution of the classical two-ray model is also presented in this graph. We aim to demonstrate the difference in results when the effects of atmospheric refractivity are considered by radiopropagation techniques. The Free Space Loss represents the results obtained when only the contribution of the direct ray path is considered. Additionally, it is possible to observe that the obtained results of the modified RT and the SSPE method have a similar behavior.

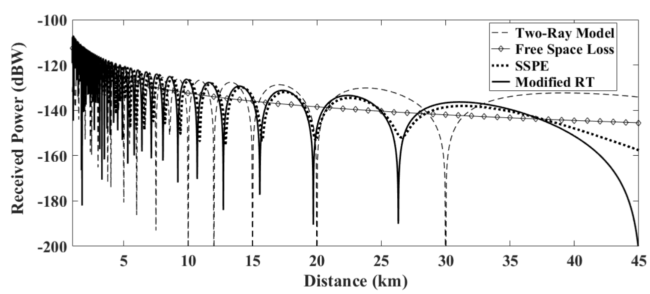

Fig. 3. Received power results for the case of a constant refractivity gradient in the entire range of distances ($\delta = -40$ N/km).

Fig. 4 exhibits the results with a change of refractivity gradients, $\delta_1 = -40$ N/km defined in the range 0-15 km, and $\delta_2 = -100$ N/km from a distance $d = 15$ km, remaining up to 60 km. It is possible to notice that at greater distances from the transmitter the results between modified RT and SSPE begin to disagree. The maximum and minimum peaks of the curve obtained with RT are formed by the reflections that are still calculated at these distances, while the SSPE numerical technique shows a smooth fading of signal strength. Analyzing the received power curve obtained with modified RT, it can be inferred that its behavior reveals the errors induced, especially at greater distances, when using the RT analytical approach. These errors are mainly due to the limitations of the proposed formulation, which is not well-suited to handling abrupt variations in the refractivity gradient between adjacent regions.

In order to consistently employ our modified RT technique, it is recommended that this approach be applied to scenarios with smooth variations in refractivity gradients between adjacent layers [10]. This is because the change in ray direction at the boundary of two adjacent layers is omitted, meaning that there are small elevation angles resulting in minor changes in the refractive index at the boundary. Then the following conditions are proposed: $\delta_1 = -40$ N/km, $\delta_2 = -45$ N/km and $\delta_3 = -50$ N/km, performing the refractivity gradient changes at $d_1 = 15$ km and $d_2 = 30$ km. Fig. 5 illustrates the ray paths conformation to calculate the received power using the Modified RT algorithm for this simulation conditions. In Fig. 5, the three regions (Region I, II and III) can be observed, related to the corresponding horizontal stratification of refractivity, where each region corresponds to the gradients δ_1, δ_2 and δ_3, as mentioned before. The blue rays represent the direct ray paths at a known receiver point. The light blue, green and red rays represent the set of incident-reflected ray paths that are conformed (based on the formulation of Section II) in order to hit at a known receiver point located in Region I, II and III, respectively.

In this way, in Fig. 6 the received power 2D distribution obtained with SSPE technique is depicted. Finally, Fig. 7 shows the results of received power comparing both techniques. This graph illustrates that, by applying smoother variations in the refractivity gradient, the received power curve obtained with RT attenuates more gradually with distance, without showing interference peaks with rapid oscillations, as seen in Fig. 4 for the same technique. This gradual attenuation of the received power with distance is expected. Moreover, since the comparison is made with SSPE (an approximate technique that propagates its numerical solution in a single principal direction, the x-axis in our work) and in a very specific case of non-homogeneous atmospheric conditions, the results shown are promising.

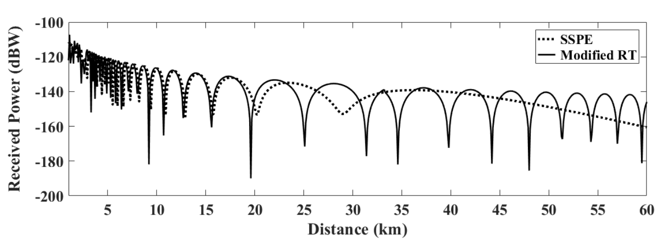

Fig. 4. Received power results for the change of gradients $\delta_1 = -40$ N/km, $\delta_2 = -100$ N/km at $d = 15$ km.

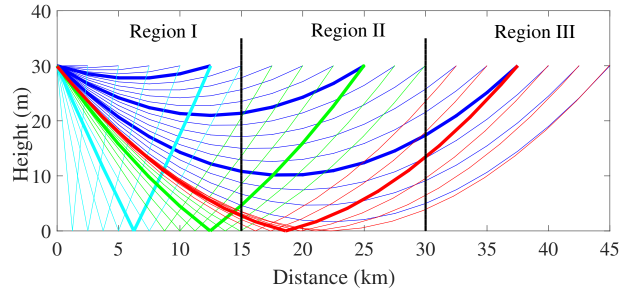

Fig. 5. Ray paths conformation to calculate received power for a refractivity horizontal variation case.

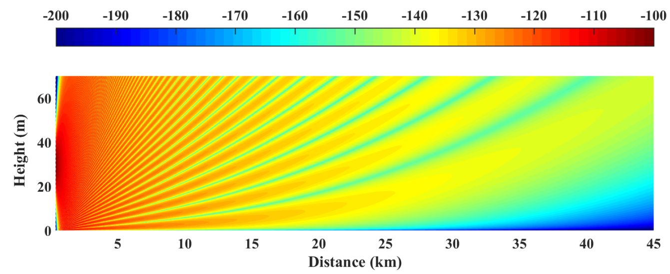

Fig. 6. Received Power (dBW) versus distance-height achieved with SSPE for a refractivity horizontal variation case at 10 GHz.

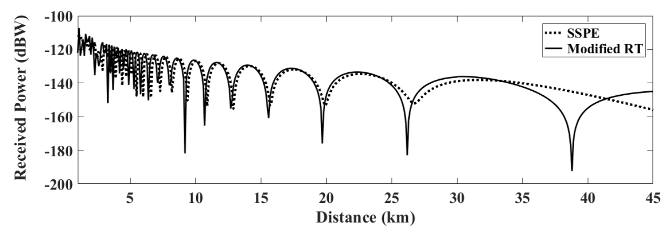

Fig. 7. Received power results for the change of gradients $\delta_1 = -40$ N/km, $\delta_2 = -45$ N/km, $\delta_3 = -50$ N/km at $d_1 = 15$ km, $d_2 = 30$ km.

IV. Conclusion

This work presents the derivation of an innovative formulation introduced to incorporate ground-reflection analysis within a multipath model based on RT solutions. Specifically, this formulation has been developed to address horizontally stratified variations in the refractivity gradient in long-distance scenarios. Simulations provided preliminary results obtained in canonical cases, which were compared with the PE numerical solution for validation. The results are promising when considering smooth variations in the refractivity gradient between adjacent regions in a horizontally stratified environment. These initial findings suggest the potential applicability of this approach to realistic radiopropagation scenarios, including other non-homogeneous conditions such as irregular terrain profiles and varying surface boundary conditions.

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