Design and Analysis of a Non-Uniform Antenna Array for IoT Applications

Anderson S. Soares, José Carlos. A. Santos

***Abstract*—This letter proposes a mathematical model that allow to analyze the total field irradiated by a non-uniform antenna array, in which the elements consist of different types, with different radiation pattern, positioned and oriented randomly in space. The results obtained are compared with the Multilevel Fast Multipole Method. In the analysis performed, it was observed that both for azimuth and elevation variation the results obtained were significant. Furthermore, the analyzed scenarios are related antennas for internet of things applications, bringing the analysis of the proposed model to practical applications. This analysis can lead to a huge range of applications, depending on interest and necessity, given the possibility of obtaining the radiation pattern formed by a non-uniform antenna array with accurately and with reduced computational resources.**

***Index Terms*—Antennas, Internet of Things, Communications Systems.**

I. Introduction

THE Internet of Things (IoT) refers to a technological revolution that aims to connect items to the internet. This promise generates enormous interest in exploring future IoT applications such as smart cities, home automation, wearable electronics, environmental monitoring, industrial IoT, and the Internet of Vehicles. Due to the development of applications focused on IoT, antennas have taken a leading role because it is through them that the conversion of an electrical signal into an electromagnetic wave is made for the purpose of transmitting and receiving information with high gain and multi-frequency characteristics [1], [2], [3].

In order to increase the directivity of an antenna, several irradiating elements can be grouped in a geometric configuration which is called antenna array [4], [5]. Antenna array have such significant advantages, including rapid reconfiguration, multibeam, sidelobe control, and high reliability over other types of antennas, and there has been an increased interest in their use for a wide variety of communication for IoT applications. Despite the initial studies on the theory of antenna array dating from the decade of 40, there are some limitations that still lack research and deepening [6], [7], [8].

As a matter of convenience, allied to mathematical complexity in characterizing the resultant electric field, antenna arrays are basically formed by identical elements, being constituted mostly by elementary antennas, isotropic antennas, dipoles, horns. Another factor is related to the geometry formed, the published studies approach only the positioning of the elements on the most common geometries, with the antennas positioned in a linear, rectangular or circular geometry [9], [10], [11], [12].

Over the last decade, several approaches based on statistical methods have been proposed to obtain the radiation pattern formed by a non-uniform antennas array. Closed analytical expressions, which take into account Uniform and Gaussian probability distributions for various types of geometric configurations and distance between the radiating elements were presented [13], [14], [15]. Another field of research related to the topic is the use of optimization algorithms for the synthesis of the problem. Such methods are based on the determination of the positioning of the elements to obtain a certain minimum level for the secondary lobes, half-power angle, as well as the direction of nulls of the radiation diagram formed by the array. Among the main techniques used, stand out Particle Swarm Optimization (PSO) and the Genetic Algorithm (GA) [16], [17], [18], [19].

The main contributions of the research stand out that the proposed approach is the only deterministic analysis technique available in the scientific literature that allows evaluation the five factors that controlling the radiation pattern formed by antennas array: the geometric, distance between the antennas, the individual radiation pattern, amplitude and phase.

II. Mathematical Model

A. Field irradiated in the far-field region by an singular antenna

Assuming that the electric field vector varies harmoniously over time and that the frequency of operation of a given antenna is known, in the far-field region, the electric field vector can be completely specified, for any polarization, through the knowledge of three components [20]:

1) Zenith component E_θ as a function of r, θ and ϕ;
2) Azimuth component E_θ as a function of r, θ and ϕ;
3) Phase difference between components E_θ and E_ϕ as a function of θ and ϕ.

Admitted that the contribution of the radial component of the electric field vector (E_r) is very small in the far-field region, the field becomes

$$\vec{E}(r,\theta,\phi) = [E_\theta(r,\theta,\phi)a_\theta + E_\phi(r,\theta,\phi)a_\phi]I_0\frac{e^{-j(kr+\delta)}}{r} \quad (1)$$

Eq. 1 establishes the solution to an infinitesimal current element positioned at the origin of the coordinate system as a

Anderson S. Soares is with the Post-graduate Program in Electrical Engineering, Military Institute of Engineering, IME – Rio de Janeiro - Brazil. E-mail: anderson.soares@marinha.mil.br. Orcid: 0000-0001-9701-0819.

José Carlos A. Santos is with the Post-graduate Program in Electrical Engineering, Military Institute of Engineering, IME – Rio de Janeiro - Brazil. E-mail: araujo@ime.eb.br. Orcid: 0000-0002-6044-3431

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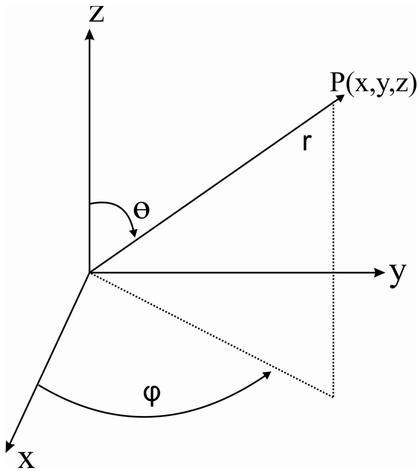

Fig. 1. reference system.

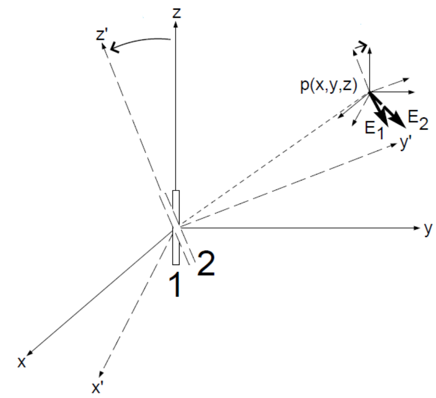

Fig. 2. Rotation effect of an antenna. Condition 1 there is a field E1 at point P (x, y, z). Condition 2, intent checking the generated field E2 at the same point in space due to the lag produced on each coordinate axis.

spherical wave spreading radially with a phase $e^{-j(kr+\delta)}$ and amplitude decaying by the factor $1/r$[21]. The magnitudes of both components of the electric field, E_θ and E_ϕ , vary inversely with the distance to the source. r is the distance between the source and the point of interest, θ the zenith angle and ϕ the azimuth angle, which are understood in the following range: $r \in [0 : \infty]$, $\theta \in [0 : \pi]$, $\phi \in [0 : 2\pi]$. k i the propagation constant in the free space. In addition, the equation relates to the relative intensity of the power supply, being given by a complex number, which has a module I_0 and phase δ, Fig. 1 shows the reference system employed.

The equation is written in spherical coordinates. In some cases, it is convenient to handle the aforementioned equations in Cartesian coordinates. Any vector field can be converted between cartesian and spherical coordinates by the expression

$$\vec{E} = E_x a_x + E_y a_y + E_z a_z = E_\rho a_\rho + E_\theta a_\theta + E_\phi a_\phi \quad (2)$$

The unitary cartesian vectors are related to the unitary spherical vectors by the following transformation matrix

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\phi & \sin\theta & 0 \end{pmatrix} \times \begin{pmatrix} a_\rho \\ a_\theta \\ a_\phi \end{pmatrix} \quad (3)$$

the matrix can be expand as follows

$$\begin{aligned} a_x &= \sin\theta\cos\phi a_\rho + \cos\theta\cos\phi a_\theta - \sin\phi a_\phi \\ a_y &= \sin\theta\sin\phi a_\rho + \cos\theta\sin\phi a_\theta + \cos\phi a_\phi \\ a_z &= \cos\phi a_\rho - \sin\theta a_\theta \end{aligned} \quad (4)$$

relating the terms obtained in Eq. 1 and Eq. 4,

$$\begin{aligned} \vec{E}(x,y,z) =& (\cos\theta\cos\phi E_\theta - \sin\phi E_\phi) I_0 \frac{e^{-j(kr+\delta)}}{r} a_x + \\ & (\cos\theta\sin\phi E_\theta + \cos\phi E_\phi) I_0 \frac{e^{-j(kr+\delta)}}{r} a_y - \\ & (\sin\theta E_\theta) I_0 \frac{e^{-j(kr+\delta)}}{r} a_z \end{aligned} \quad (5)$$

putting the terms in evidence,

$$\vec{E}(x,y,z) = I_0 \frac{e^{-j(kr+\delta)}}{r} [(\cos\theta\cos\phi E_\theta - \sin\phi E_\phi) a_x + (\cos\theta\sin\phi E_\theta + \cos\phi E_\phi) a_y - \sin\theta E_\theta a_z] \quad (6)$$

Eq. 6 establishes the electric field irradiated by an antenna positioned at the origin, for a given point in the far-field region, due to the use of Cartesian's Coordinates.

B. Field irradiated by an antenna positioned at origin arbitrarily oriented in space

It is intended to obtain the resultant electric field vector for a composition of antennas with different spatial orientations. It is considered the case where a given antenna is positioned at the origin of the coordinate system, position 1, generating a field E_1 at the point $P(x,y,z)$. Making a change of orientation from position 1 to position 2 and check the contribution of the electric field E_2 at the same point $P(x,y,z)$, Fig. 2

In three-dimensional space, a point of coordinates $P(x,y,z)$ can be rotated around one of the coordinated axes by multiplying for an array called the rotation matrix [22]

$$\begin{pmatrix} E_{xrotated} & E_{yrotated} & E_{zrotated} \end{pmatrix} = \begin{pmatrix} matrix \end{pmatrix} \times \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \quad (7)$$

The rotation matrix can be obtained by the unitary vectors when rotated around an axis. The method uses an array 3 × 3 to perform the proper rotations. A rotation over the shaft x is given by the following matrix, where the angle α is related to the lag of interest on the axis x

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \quad (8)$$

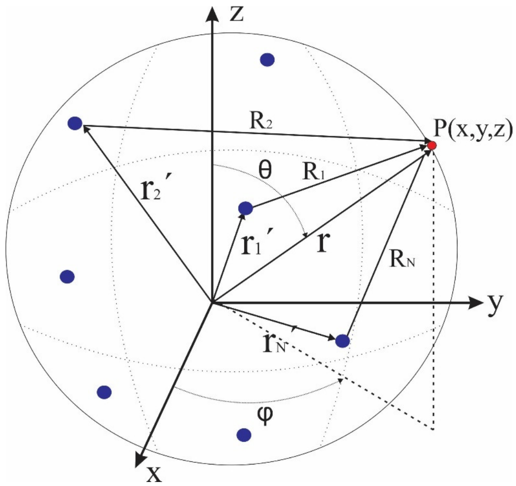

Fig. 3. Geometric illustration of a non-uniform antennas array with N elements randomly positioned in space. Blue symbol represents a randomly positioned antenna in space. In red, the distant field region.

a rotation over the shaft y is given by, in which the angle β is related to the lag of interest on the axis y

$$\begin{bmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{bmatrix} \tag{9}$$

a rotation over the axis z is given by, in which the angle is related to the lag of interest on the axis z

$$\begin{bmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix} \tag{10}$$

C. Field irradiated by a non-uniform antenna array

Previously, was defined as the electric field irradiated vector for a single element. This section expanded the analysis to several elements. Consider a spherical surface with center at the origin, where the antennas are randomly distributed in space, $r_1'(x_1, y_1, z_1)$, $r_2'(x_2, y_2, z_2)$,...,$r'_N(x_N, y_N, z_N)$. Assuming that intends to get the resultant electric field vector at the point $P(x, y, z)$ seen by an observer positioned at the origin, the vector modules distances formed between the radiant elements and the observation point are given by R_1, R_2,..R_N, Fig. 3

The general solution can be obtained by superposition of elementary spherical waves with expressions given by Eq. 6, the vector being the total electric field given by

$$\vec{E}_{result} = \vec{E_1} + \vec{E_2} + ... + \vec{E_N} \tag{11}$$

each term in Eq. 11 refers to complex numbers. The vector sum must be performed in Cartesian coordinates. The resulting field is given by

$$\begin{aligned}\vec{E}(r) =& I_1\frac{e^{-j(kR_1+\delta_1)}}{R_1}[(\cos\theta\cos\phi E_{\theta 1} - \sin\phi E_{\phi 1})a_x\\ &+ (\cos\theta\sin\phi E_{\theta 1} + \cos\phi E_{\phi 1})a_y - \sin\theta E_{\theta 1}a_z]+\\ &I_2\frac{e^{-j(kR_2+\delta_2)}}{R_2}[(\cos\theta\cos\phi E_{\theta 2} - \sin\phi E_{\phi 2})a_x+\\ &(\cos\theta\sin\phi E_{\theta 2} + \cos\phi E_{\phi 2})a_y - \sin\theta E_{\theta 2}a_z]+,\dots,+\\ &I_N\frac{e^{-j(kR_N+\delta_N)}}{R_N}[(\cos\theta\cos\phi E_{\theta N} - \sin\phi E_{\phi N})a_x+\\ &(\cos\theta\sin\phi E_{\theta N} + \cos\phi E_{\phi N})a_y - \sin\theta E_{\theta N}a_z]\end{aligned} \tag{12}$$

the phase of each element ($e^{-jkR_i+\delta_i}$) will oscillate many cycles until it reaches the observation point. Any simplification in this term will result in an error in the calculation of the resulting field. However, it is possible to achieve an approximation in the amplitude factor, because in the far-field region $r \approx \vec{R}_1 \approx \vec{R}_2 \approx \vec{R}_N$

$$\begin{aligned}\vec{E}(r) =& \frac{1}{r}\{I_1 e^{-j(kR_1+\delta)}[(\cos\theta\cos\phi E_{\theta 1} - \sin\phi E_{\phi 1})a_x+\\ &(\cos\theta\sin\phi E_{\theta 1} + \cos\phi E_{\phi 1})a_y - \sin\theta E_{\theta 1}a_z]+\\ &I_2 e^{-j(kR_2+\delta)}[(\cos\theta\cos\phi E_{\theta 2} - \sin\phi E_{\phi 2})a_x+\\ &(\cos\theta\sin\phi E_{\theta 2} + \cos\phi E_{\phi 2})a_y - \sin\theta E_{\theta 2}a_z]+,\dots,+\\ &I_N e^{-j(kR_N+\delta)}[(\cos\theta\cos\phi E_{\theta N} - \sin\phi E_{\phi N})a_x+\\ &(\cos\theta\sin\phi E_{\theta N} + \cos\phi E_{\phi N})a_y - \sin\theta E_{\theta N}a_z]\}\end{aligned} \tag{13}$$

rewriting the terms according to a summation,

$$\begin{aligned}\vec{E}(r) =& \frac{1}{r}\sum_{i=1}^{N}[(\cos\theta\cos\phi E_{\theta i} - \sin\phi E_{\phi i})a_x + (\cos\theta\sin\phi E_{\theta i}+\\ &+ \cos\phi E_{\phi i})a_y - \sin\theta E_{\theta i}a_z]I_i e^{-j(kR_i+\delta_i)}\end{aligned} \tag{14}$$

Eq. 14 establishes the total field irradiated by a non-uniform antennas array positioned randomly in space. It is worth noting that the aforementioned equation is valid for antennas array that cover several special cases, such as:

1) antennas with different radiation patterns, because the terms $E_{\theta i}$ and $E_{\phi i}$, typically, represent values of the magnitude of the electric field as a function of spatial coordinates. Therefore, nothing prevents individual elements from using field diagrams that are not identical;
2) it can also be considered that the individual elements are excited by different amplitudes I_i and phases σ_i;
3) the equation obtained is valid for any geometric configuration. The elements can be arranged linearly on a specific axis or in a plane because the term R_i do not impose any restriction on the positioning of the antennas.

III. NUMERICAL RESULTS

After obtaining the equation that describes the total field irradiated by a non-uniform antenna array, positioned randomly in space, simulations are performed to form to evaluate the proposed mathematical model. For comparison purposes, it is considered a practical case of use. In this way, a hypothetical

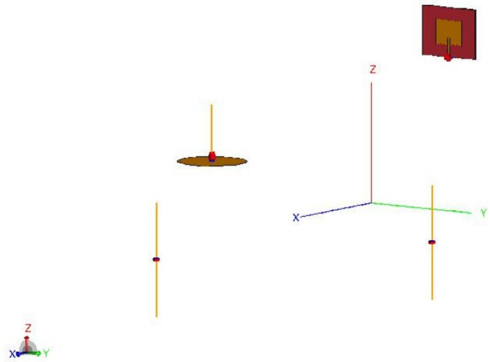

Fig. 4. Geometric illustration of a non-uniform array formed by four differents antennas positioned in the FEKO software for simmulation.

scenario is assembled in order to evaluate the efficacy of the model proposed by equation 14 for more complex cases, which consist a non-uniform antenna array with different radiation patterns: two dipoles of half wavelength; a quarter-wave monopole and a path antenna. The assembled scenario refers to a possible application of IoT in which several devices, with different antennas, need to communicate with a service provider forming an antenna array to collect and exchange data for a wide range of life applications such as industry, transportation, logistics, healthcare, smart environment and city information.

The proposed mathematical model was compared with the Multilevel Fast Multipole Method (MLFMM), which is used for electromagnetic calculation [23]. The software used was CAD-FEKO [24]. The geometric characteristics of the antennas were chosen to operating at 2.4 GHz center frequency, Fig. 4. In possession of the electrical field information generated for the antennas, as well as the respective positions, the simulations were performed in order to assess the degree of reliability of the proposed method. Tab 1. presents the steps of executing an algorithm used to simulated:

TABLE I
ALGORITHM USED TO SIMULATED THE EQU. 14 AND MLFMM METHOD.

START
1: **Inputs**{f, N, R, I, , x, y, z}
2: **Initialize** Ex= 0; Ey= 0; Ez= 0
3: **Compute** propagation constant in the free space
4: **For** i = 1 to N **do**
5: **Read** $E_{\theta i}, E_{\phi i}$
6: **Compute** the distance vector module
7: **Compute** $(\cos\theta_i \cos\phi_i E_{\theta i} - \sin\phi_i E_{\phi i}) I_i e^{-j(kR_i+\delta_i)}$
8: **Compute** $(\cos\theta_i \sin\phi_i E_{\theta i} + \cos\phi_i E_{\phi i}) I_i e^{-j(kR_i+\delta_i)}$
9: **Compute** $(-\sin\theta_i E_{\theta i}) I_i e^{-j(kR_i+\delta_i)}$
10: **End**
11: **Convert** the result of lines 7, 8 and 9 to spherical coordinates
12: **Compute** the module of the result obtained in the line 11
13: **Plot** the result obtained as a function of phi and theta
END

Fig. 5 and Fig. 6 presents the final result obtained. Despite the complexity of the model under analysis, it is observed that the results obtained both in azimuth and elevation variation are close to the results generated by MLMMF. Moreover, it is important to mention that the processing time and allocation of computational resources required by the proposed method are substantially lower than the software used for comparison, Tab. II.

TABLE II
TIME AND COMPUTATIONAL RESOURCES REQUIRED.

	MLFMM	**Model Proposed**
Mesh (amount)	7172	0
Memory allocation (Gbyte)	6,50	1,23
Time (seconds)	123,75	3,54

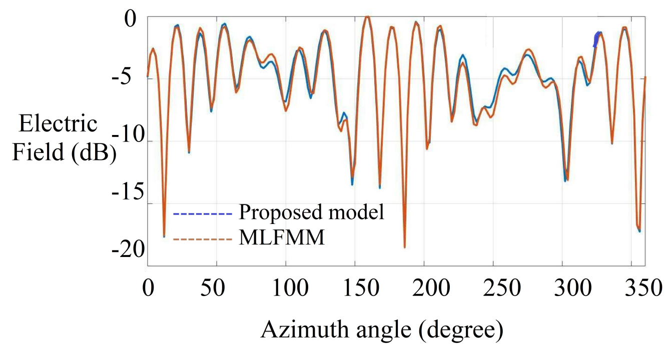

Fig. 5. Azimuth diagram formed by the antennas in figure 4. In blue, the results from the proposed model; in yellow, the results from MLFMM method.

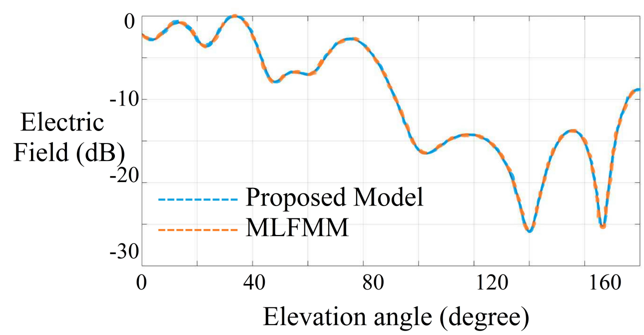

Fig. 6. Elevation diagram formed by the antennas in Fig. 4. In blue, the results from the proposed model; in yellow, the results from MLFMM method.

IV. CONCLUSION

This letter describes a mathematical model that allows characterizing the electric field formed by a non-uniform antennas array, which consider antennas of different types, with different radiation pattern, positioned and randomly oriented towards a particular reference center. In the analysis performed, it is observed that both for azimuth and elevation variation the results obtained were close to the expected.

The scenario under study is associated with a typically communications in the Internet of Things, transporting the analysis of the proposed model for possible applications. The scenario presented is hypothetical, since supposedly the antennas in question are part of the same array. Usually, the antennas on these platforms are independent, corresponding to different communications systems. In any way, this analysis opens up space for a huge range of applications, depending on the interest and necessity. Therefore, the proposed model can become a great tool for the treatment of antennas, in order to obtain the electric field formed by a non-uniform antenna array.

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