JOURNAL OF COMMUNICATION AND INFORMATION SYSTEMS, VOL. 38, NO. 1, 2023.

120

Combined Polar Codes and 2-12-QAM Applied to 5G Communications

Flávia Camila Morais de Oliveira, Maria de Lourdes Melo Guedes Alcoforado, Garik Markarian, Valdemar Cardoso da Rocha Jr. and Waslon Terllizzie Araújo Lopes

Abstract—The 5th generation of cellular communications systems (5G), standardized by the 3rd Generation Partnership Project (3GPP), uses polar codes in both physical download control channel (PDCCH) and physical download shared channel (PDSCH), combined with some low order modulations, such as QPSK and 16-QAM. Recently, 2-12-QAM modulation was proposed as an energy-efficient scheme when compared to 16-QAM. This article investigates the performance of a communication system using polar codes combined with a 2-12-QAM modulation scheme over channels disturbed by additive white Gaussian noise (AWGN) as well as over channels disturbed simultaneously by Rayleigh fading and AWGN. The 2-12-QAM modulation is compatible with legacy 16-QAM still widely used, and when combined with appropriate error correcting codes produces results that approach the Shannon limit.

Index Terms—Polar Codes, Quadrature Amplitude Modulation (QAM), 5G, Rayleigh Fading Channel.

I. INTRODUCTION

QUADRATURE amplitude modulation (QAM) has been included in current wireless communication standards due to its high bandwidth efficiency [1]. The 16-QAM constellation with Gray code mapping performs well and in combination with Low Density Parity Check (LDPC) codes or Turbo codes it can approach the Shannon limit [2], [3]. High power linear amplifiers are required at the transmitter side for some practical applications employing QAM modulation [4], however due to non-linear distortions caused by saturation of the power amplifiers, the Shannon limit is far from being reached [5]. Various linearization techniques [6], [7] have been introduced to reduce the non-linear effect, but changing hardware makes implementation difficult due to non-compatibility with systems already in use [8].

The recently proposed multidimensional 2-12-QAM modulation [8] presents relevant energy gains using just software

F. C. M. Oliveira and M. L. M. G. Alcoforado are with the Postgraduate Program in Systems Engineering, Polytechnic School of Pernambuco, University of Pernambuco, Recife, PE, Brazil. E-mails: fcmo@poli.br and mlmga@poli.br.

V. C. Rocha Jr is with CODEC, Department of Electronics and Systems, Federal University of Pernambuco, Recife, PE, Brazil. E-mail: valdemar.rochajr@ufpe.br.

G. Markarian is with RINICOM LT, Lancaster, UK. E-mail: garik@rinicom.com.

W. T. A. Lopes is with the Department of Electrical Engineering, Federal University of Paraíba, João Pessoa, PB, Brazil. E-mail: waslon@ieee.org.

mapping changes in the existing 16-QAM modulation. The 2-12-QAM modulation scheme uses two 16-QAM constellations, and eliminates the four highest energy symbols in each 16-QAM constellation to reduce the peak-to-average power ratio (PAPR). The 2-12-QAM maps a 7 bit binary pattern into a 4-dimensional symbol.

In order to reduce the effect of noise in system performance, error correcting codes are used which add redundant bits to the information sequence before transmission in order to improve reliability [9]. The good error correction capability of polar codes allowed them to be selected by the 3rd Generation Partnership Project (3GPP) as the code for both physical download control channel (PDCCH) and physical download shared channel (PDSCH), the control channel in the 5th generation (5G) of mobile communication systems [10], [11]. It is worth to mention that polar codes enjoy the practical advantage of admitting low complexity encoding and decoding algorithms [12].

In usual practice, the block length of a polar code is a power of 2, therefore not being immediately compatible for combining with some modulation schemes, such as 2-12-QAM, for example. In this case, a kernel matrix of adequate size combined with a standard kernel matrix is recommended [13]. Another alternative to address the compatibility issue is the Arikan punctured kernel matrix [14]. However, both solutions require a high computational complexity. Another solution, which consists of appending a suffix of zeros to each codeword, can be employed in order to reduce computational complexity [15].

A polar code is designed to operate efficiently in a specific channel, and thus it is a challenge to use polar codes in a practical situation where the communication channel changes with time, as occurs in the presence of fading during transmission in free space [16]. The problem presented by the use of polar codes over fading channels is addressed in [17], considering an additional constraint on the permissible average and peak power. Their model reduces the fading channel to a model consisting of a cascade of an AWGN channel followed by an erasure channel.

In this article we apply polar codes to a fading channel without providing any further mitigation in order to cope with fading conditions. Our goal is to analyze the performance of polar codes combined with 2-12-QAM modulation in AWGN channels and Rayleigh fading channels disturbed by AWGN, when compared to legacy 16-QAM modulation under similar noise conditions, and discuss the results obtained by computer simulation.

The remaining sections of this article are organized as follows. In Section II, the 2-12-QAM modulation is described. A review of polar codes is presented in Section III. The proposed system model is discussed in Section IV. The simulated results are presented in Section V. Finally, Section VI is devoted to the conclusions.

II. 2-12-QAM MODULATION

Fig. 1 illustrates a scheme used for data transmission in several communication systems employing a rate $3/4$ error correcting code, a 16-QAM modulator and a power amplifier (PA). The error correcting code is often referred to as a forward error correcting (FEC) code [9]. The system illustrated by the block diagram in Fig. 1 has a significant drawback as the power amplifier operates with an average power far away from its maximum power [18], in order to avoid non-linear distortions. Aiming to operate effectively, a power margin is required from the power amplifier with sufficient power to feed the 16-QAM modulation when highest energy symbols occur. This leads to a significant reduction in the overall system efficiency [19]. Several techniques have been introduced in order to reduce this undesirable effect, such as non-linearity correction and reduction of PAPR, among others [6], [7], [20], [21]. However, significant changes are required to overcome this negative effect and, because of that, questions about compatibility with current systems were raised.
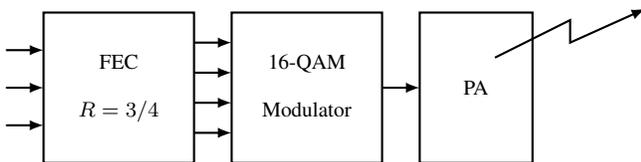
Fig. 1. Block diagram of a transmitter for a 16-QAM system with a rate $3/4$ FEC code.

In order to use the 2-12-QAM modulation to replace an existing 16-QAM in a transmission system like the one indicated in Fig. 1, it was proposed in [8] the transmission system illustrated in Fig. 2, consisting of a rate $6/7$ FEC code, a 2-12-QAM modulator and a PA. The 2-12-QAM modulation is based on the legacy 16-QAM modulation with mapping changes and offers energy gain and compatibility with current systems [8].

In a 16-QAM system [22], a signal $s_i(t)$, $1 \leq i \leq 16$, $0 < t \leq T$, received in a time slot T is given by

$$s_i(t) = A_i \cos(2\pi f_c t) + B_i \sin(2\pi f_c t), \qquad (1)$$

where A_i and B_i are the amplitudes corresponding to axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, and f_c denotes the carrier frequency. The corresponding 16-QAM constellation is illustrated in Fig. 3, and this format consists of 16 two-dimensional vectors, denoted as a set by $\{[A_i, B_i]\}$, where $i \in \{1, \ldots, 16\}$. The amplitudes A_i and B_i take independent values in the set $\{\pm 1, \pm 3\}$ which causes the distance between adjacent symbols to be 2. In general, the amplitudes independently take values from the set $\{\pm 1, \pm 3, \ldots, \pm \sqrt{M} - 1\}$, where M denotes the size of the constellation, and the

separation between adjacent symbols is denoted by $2d$, where the parameter d is related to the signal energy [22].
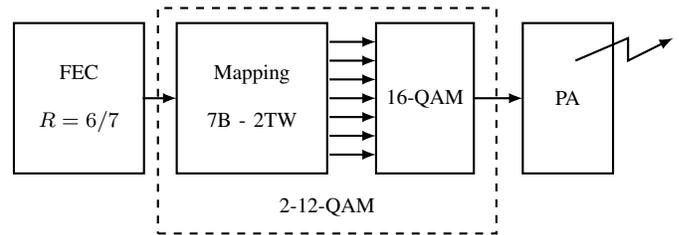
Fig. 2. Block diagram of a transmitter for a 2-12-QAM system with a rate $6/7$ FEC code.
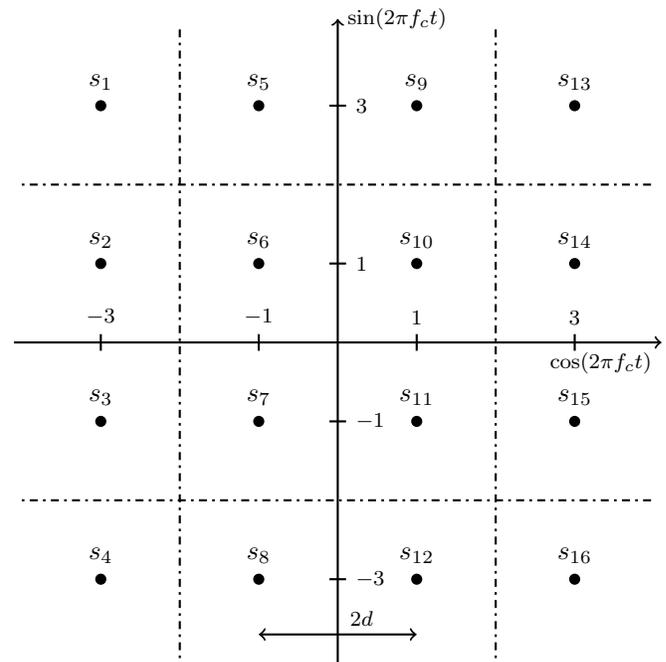
Fig. 3. Diagram of 2-dimensional 16-QAM constellation.

The 2-12-QAM is a multidimensional modulation obtained by eliminating the four highest energy symbols from two two-dimensional 16-QAM constellations similar to the one depicted in Fig. 3. The eliminated symbols are those indicated by s_1, s_4, s_{13} and s_{16}, situated in the regions marked with a red sign in Fig. 4. As a result we have two constellations with twelve symbols each (2-12-QAM) to be mapped into a 4-dimensional signal.

The 16-QAM modulation combined with a rate $3/4$ FEC has spectral efficiency of 3 bits/symbol [23]. In order to maintain the same spectral efficiency in a 2-12-QAM system, a rate $6/7$ FEC and a new mapping is employed, by which a 7 bit block is associated to two 12-ary symbols as indicated in Fig. 2, by the block labeled as 7B - 2TW mapping. One advantage of this format is the possibility of using a 16-QAM modulator/demodulator hardware and obtain 2-12-QAM modulation through a software update [8].

As explained in [8], a 2-12-QAM signal $s_{ij}(t), 0 < t \leq 2T$,
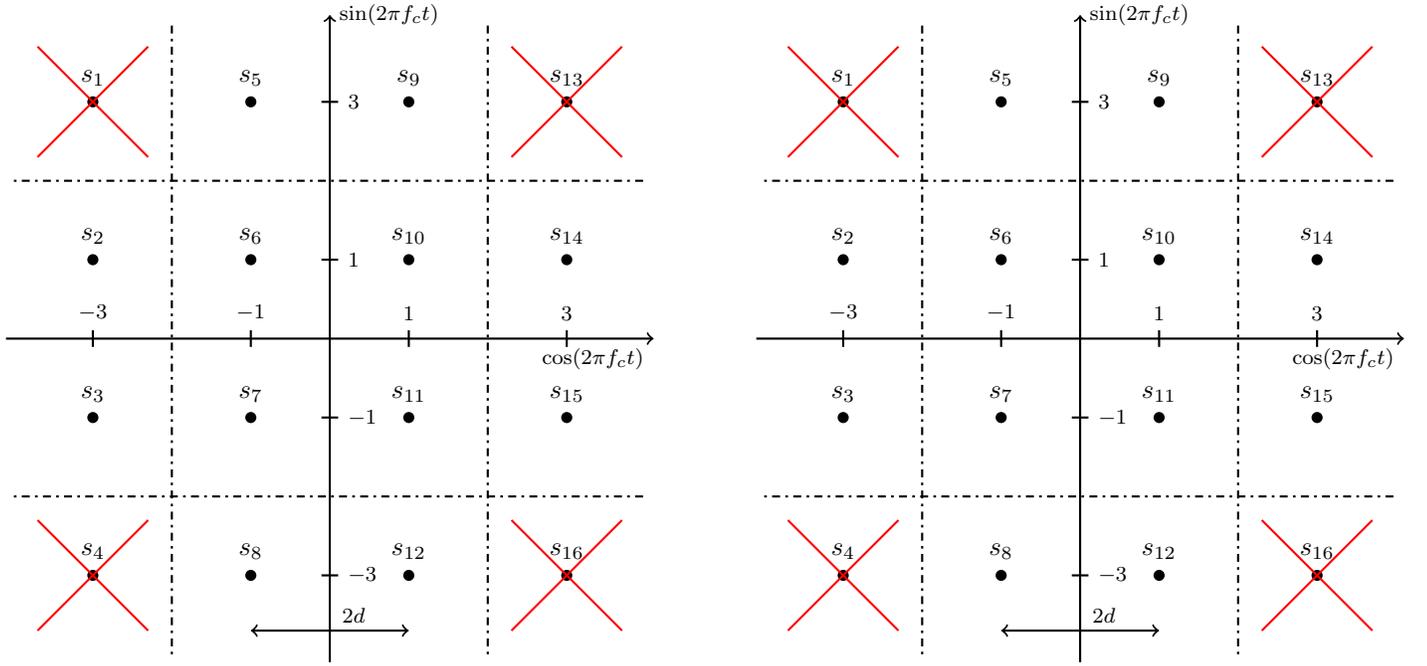
Fig. 4. Diagram of 4-dimensional constellation consisting of two 12-QAM constellations, thus forming a 2-12-QAM constellation.

received in time slot $2T$ is represented as

$$s_{ij}(t) = \begin{cases} A_i \cos(2\pi f_c t) + B_i \sin(2\pi f_c t), & 0 < t \le T \\ C_j \cos(2\pi f_c t) + D_j \sin(2\pi f_c t), & T < t \le 2T, \end{cases}$$
(2)

where the indices i and j are each associated with a distinct constellation in Fig. 4, and i and j independently take values in the set $\{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$. A_i and B_i are amplitudes allowed in the axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, for one constellation, and C_j and D_j are amplitudes allowed in the axes $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$, respectively, in the other constellation in Fig. 4.

In order to map bit blocks to constellation symbols we require the number of constellation symbols to be a power of 2. For the 2-12-QAM constellation, in order to reduce the number of symbols from 144 to 128, which is a power of 2, we further eliminate 16 symbols, i.e., 16 (i,j) forbidden pairs, chosen from among the 64 higher energy symbols associated with the constellation, where i and j take values in the set $\{2, 3, 5, 8, 9, 12, 14, 15\}$. Thus, the signal constellation of 2-12-QAM after pruning consists of 128, i.e., 2^7, 4-dimensional vectors. Each such vector is one-to-one associated to 7 information bits and is denoted as a set by $\{[A_i, B_i], [C_j, D_j]\}$, where i and j take values in the set $\{2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15\}$ under the constraint of not constituting an (i,j) forbidden pair. The amplitudes A_i, B_i, C_j and D_j take independent values in the set $\{\pm 1, \pm 3\}$ which causes the distance between adjacent symbols to be 2 in each constellation. In general, the amplitudes independently take values from the set $\{\pm 1, \pm 3, \dots, \pm \sqrt{M} - 1\}$, where M denotes the size of each constellation, and the separation between adjacent symbols is denoted by $2d$.

The corresponding probability density function for the received vector r after transmission over an AWGN channel is given by

$$\rho_{r/s_{i,j}}(r) = \left(\frac{1}{\pi N_0}\right)^2 \exp\left[\frac{-(r_1 - A_i)^2 + (r_2 - B_i)^2}{N_0}\right]$$
$$\times \exp\left[\frac{-(r_3 - C_j)^2 + (r_4 - D_j)^2}{N_0}\right],$$
(3)

where r_1, r_2, r_3 and r_4 are the components of the received vector r after transmission over an AWGN channel, given by

$$r = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} A_i + n_1 \\ B_i + n_2 \\ C_j + n_3 \\ D_j + n_4 \end{bmatrix},$$
(4)

where n_1, n_2, n_3 and n_4 are the zero-mean additive Gaussian noise components with variance $\sigma^2 = N_0/2$.

III. POLAR CODES

The concepts employed in this section are covered in more detail in [12]. Polar codes, proposed by Arikan [12], can achieve the symmetric capacity of binary discrete memoryless channels (B-DMC) through channel polarization. Channel polarization refers to the operation that allows synthesizing N independent copies of a B-DMC W, corresponding to N uses of W, denoted as $W_N^{(i)}, 1 \le i \le N$ [12].

Following [12], a generic B-DMC, denoted as $W : \mathcal{X} \to \mathcal{Y}$, is considered with a binary input alphabet $\mathcal{X}$, i.e., $\mathcal{X} = \{0, 1\}$, output alphabet $\mathcal{Y}$, and transition probabilities $W(y|x), x \in \mathcal{X}, y \in \mathcal{Y}$. The goal of this technique is to select the most suitable channels for transmission of K information bits, with N encoded bits, and the code rate R is defined as $R = K/N$.

The remaining $N - K$ positions in the input vector are filled with a fixed value each, known by the decoder in advance, which are called frozen bits [12]. The choice of the most suitable channels is made by the use of the Bhattacharyya parameter, defined as

$$Z(W) \triangleq \sum_{y \in \mathcal{Y}} \sqrt{W(y|0)W(y|1)}. \tag{5}$$

Considering base-2 logarithms, $Z(W)$ is within the interval $[0, 1]$. The physical channel considered in this work has Bhattacharyya parameter given by [24]

$$Z(W) = e^{-RE_b/N_0}, \tag{6}$$

in which E_b/N_0 is the signal-to-noise ratio (SNR) per bit. In order to determine the frozen bit positions, $R = 0.5$ and $E_b/N_0 = 2$ dB were used in this paper.

There are two distinct phases in this approach, namely a channel combining phase and a channel splitting phase. The first phase consists of combining N copies of W to produce the channel $W_N : X^N \rightarrow Y^N$ in a recursive manner, where $N = 2^n$, $n \geq 1$. In its first level, two copies of W_1 are combined to form the channel $W_2 : X^2 \rightarrow Y^2$ with transition probabilities

$$W_2(y_1, y_2|u_1, u_2) = W^N(y_1|u_1 \oplus u_2)W(y_2|u_2), \tag{7}$$

where u_1, u_2 are the inputs to channel W_2 and y_1, y_2 are its outputs.

The next phase, after having synthesized the channel vector W_N out of W^N, consists of splitting W_N back into a set of N channels $W_N^{(i)} : X \rightarrow Y^N \times X^{(i-1)}$, $1 \leq i \leq N$, with (y_1^N, u_1^{i-1}) as the output of $W_N^{(i)}$ and u_i as its input. The transition probabilities are given by

$$W_N^{(i)}(y_1^N, u_1^{i-1}|u_i) \triangleq \sum_{y \in \mathcal{Y}, u_{i+1}^N \in X^{N-i}} \frac{1}{2^{N-1}} W_N(y_1^N|u_1^N). \tag{8}$$

Given the bit sequence u, the codeword x is generated by the following operation

$$x = uG_N, \tag{9}$$

where G_N is the generator matrix of order N for the encoder [11], given by

$$G_N = B_N F^{\otimes n}, \tag{10}$$

where B_N is a bit-reversal permutation, $F^{\otimes n}$ is the n-th Kronecker power of F [12], and

$$F \triangleq \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}. \tag{11}$$

The construction of polar codes used in this paper is based on (9)-(11) considering the frozen bit positions obtained through (6). Other construction methods can be found in [25]–[28].

The decoding process takes place with the use of the Successive Cancelation (SC) algorithm, also proposed in [12]. In order to estimate the decoded message, an SC decoder calculates the probability of the value of a certain bit by making use of values found on the encoder structure. In case of decoding a frozen bit, the value zero (0) is automatically assigned by the decoder to that frozen bit.

IV. SYSTEM MODEL

The block diagram of the proposed system is shown in Fig. 5.
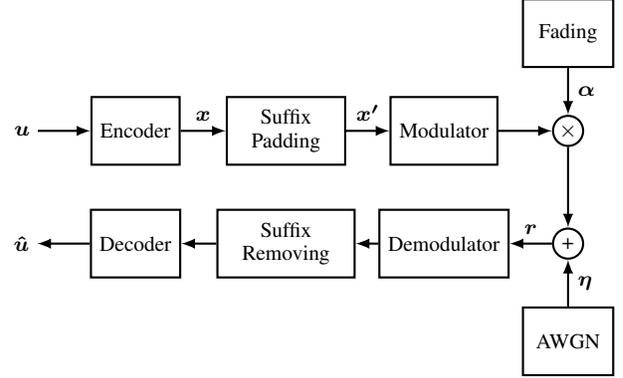
Fig. 5. Block diagram of the proposed transmission system employing coded 2-12-QAM modulation.

The idea here is to investigate the performance of the proposed system to transmit information, when polar codes are combined with 2-12-QAM modulation over AWGN channels and AWGN channels disturbed by Rayleigh fading. Initially, an information source generates a sequence u of equiprobable bits to be encoded as indicated in (9). Then, the generated codeword is split into 7-bit blocks to be mapped into 2-12-ary symbols according to the 2-12-QAM scheme described earlier. As the block length of the polar code used in this article is a power of 2, which is not an integer multiple of 7, at least in principle, it is not compatible with the 2-12-QAM modulation. In order to overcome this difficulty a possible approach is to develop a scheme that combines the standard polarization kernel with an elaborate 7×7 design kernel [12], [13]. In order to reduce the complexity of code construction and decoding algorithm in this article an uncoded suffix of zeros corresponding to $s = 7 - (N \mod 7)$ is added to each codeword [15]. Then, this modified codeword x', with suffix s, has length N_s given by

$$N_s = N + s. \tag{12}$$

After the mapping according to the 2-12-QAM scheme, the modulated sequence is transmitted through AWGN channels and Rayleigh fading channels, with transmission rate $R = K/N_s$. The channel output is denoted by the vector r_k in (13), $1 \leq k \leq N_s/7$.

$$r_k = \begin{bmatrix} r_{1k} \\ r_{2k} \\ r_{3k} \\ r_{4k} \end{bmatrix} = \begin{bmatrix} \alpha A_{ik} + n_{1k} \\ \alpha B_{ik} + n_{2k} \\ \alpha C_{jk} + n_{3k} \\ \alpha D_{jk} + n_{4k} \end{bmatrix}, \tag{13}$$

where n_{1k}, n_{2k}, n_{3k} and n_{4k} are zero-mean additive Gaussian noise components with variance $\sigma^2 = N_0/2$, A_{ik}, B_{ik}, C_{jk} and D_{jk} are the set of true hypothesis, α is the fading amplitude of the Rayleigh channel with probability density function given by [29]

$$p(\alpha) = 2\alpha e^{-\alpha^2}. \tag{14}$$
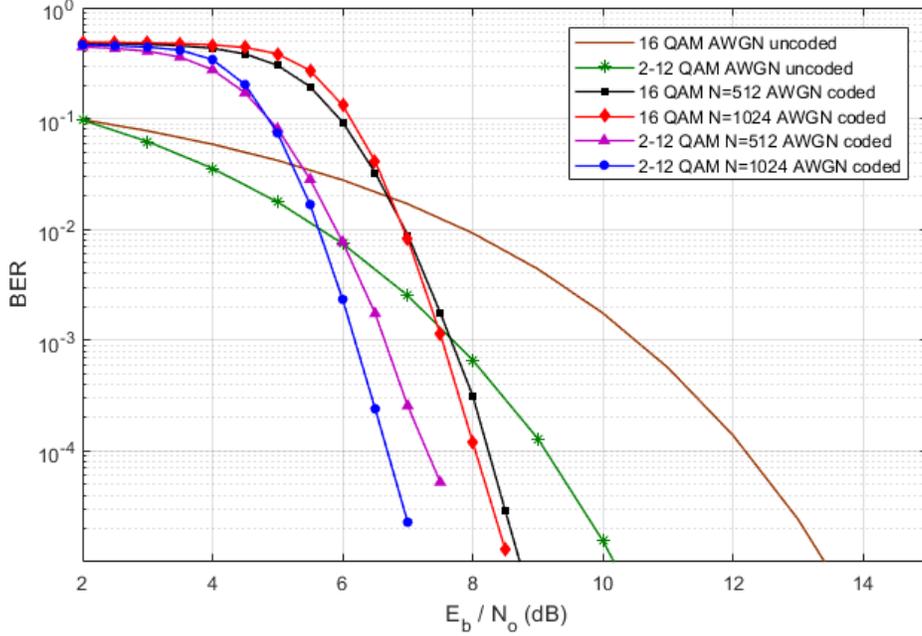
Fig. 6. BER versus E_b/N_0 performance over AWGN channels of uncoded 2-12-QAM and uncoded 16-QAM, and 2-12-QAM and 16-QAM combined with polar codes, for $N = 512$ and 1024.

Without loss of essential generality, equation (14) considers a normalized fading, that is, $E[\alpha^2] = 1$, where $E[\cdot]$ denotes the expected value operator [30].

After transmission over AWGN channels or Rayleigh fading channels disturbed by AWGN, and assuming equiprobable bits, $P(b = 0) = P(b = 1) = 1/2$, the Log Likelihood Ratio (LLR) (λ_t) for bit x_t is calculated according to [31] as follows

$$\lambda(x_t) = \ln \left(\frac{\sum_k p(r_k|\alpha, x_t = 0)}{\sum_k p(r_k|\alpha, x_t = 1)} \right), \qquad (15)$$

where $1 \leq t \leq N_s$. We consider a normalized fading and the decision rule employed is $b_t = 0$ if $\lambda \geq 0$ and $b_t = 1$ otherwise, and the probability $p(r_k|\alpha_k, x_k = b)$ can be written as (16). The last s bits provided by the 2-12-QAM demodulator are discarded and a sequence of length N is decoded through the conventional SC algorithm.

$$p(r_k|\alpha_k, x_k = b) =$$
$$\sum_{ik} \sum_{jk} \frac{1}{2\pi\sigma^2} \exp \left[-\frac{(r_{1k} - \alpha_k A_{xk})^2 + (r_{2k} - \alpha_k B_{yk})^2}{2\sigma^2} \right] \cdot$$
$$\frac{1}{2\pi\sigma^2} \exp \left[-\frac{(r_{3k} - \alpha_k C_{sk})^2 + (r_{4k} - \alpha_k D_{zk})^2}{2\sigma^2} \right] \cdot$$
$$(16)$$

V. Results and Discussion

This section presents simulation results concerning the proposed system illustrated in Fig. 5. Routines in Matlab® were implemented and the system performance was assessed using the Monte Carlo Method. For each bit error probability value, a minimum of 10,000 blocks were transmitted.

Fig. 6 shows performance curves expressed in terms of the bit error rate (BER) as a function of the E_b/N_0 for coded and uncoded 16-QAM and 2-12-QAM modulations schemes considering the AWGN channel. For a fair comparison, we consider that a system using uncoded 16-QAM modulation with Gray code mapping has an average symbol energy $E_s = \frac{160}{16}d^2 = 10d^2$. As $E_s = 4E_b$ and considering $d = 1$ without loss of the generality, $\sigma^2 \approx 1.25/10^{E_b/N_0}$, when E_b/N_0 is given in decibels (dB). In the same manner, for 2-12-QAM modulation the the average symbol energy is $\overline{E}_s = \frac{88}{12}d^2 \approx 7.33d^2$ and noise variance is $\sigma^2 \approx 0.54/10^{E_b/N_0}$. Observing the curves in Fig. 6, we notice that the system employing uncoded 2-12-QAM modulation presents an energy gain of approximately 3.1 dB for a BER of 10^{-4} when compared to the system that uses uncoded 16-QAM modulation, when both systems are operating over AWGN channels.

Next, we consider a coded communication system as indicated in Fig. 2 fed with an information sequence u of length $K = 880$ as the input to the encoder, using a polar code of rate $R = 6/7$ and block length 1024. A codeword x of length $N = 1024$ has a suffix of 5 bits added in order to be compatible with 2-12-QAM. Thus, the final codeword x' has length $N_s = 1029$. The modulated sequence passes through an AWGN channel with variance $\sigma^2 \approx 0.61/10^{E_b/N_0}$. At the receiver, after calculation of the LLRs and discarding the suffix, the SC algorithm is employed. The corresponding performance curves are shown in Fig. 6. The system using 2-12-QAM with a polar code, when compared to uncoded 2-12-QAM presents energy gains of about 1.5 dB and 2.3 dB for BER equal to 10^{-3} and 10^{-4}, respectively, with a block length $N = 1024$.

For comparison purposes, a coded system as illustrated in Fig. 1, with 16-QAM modulation is considered which employs Gray code mapping and a polar code with block length $N =$
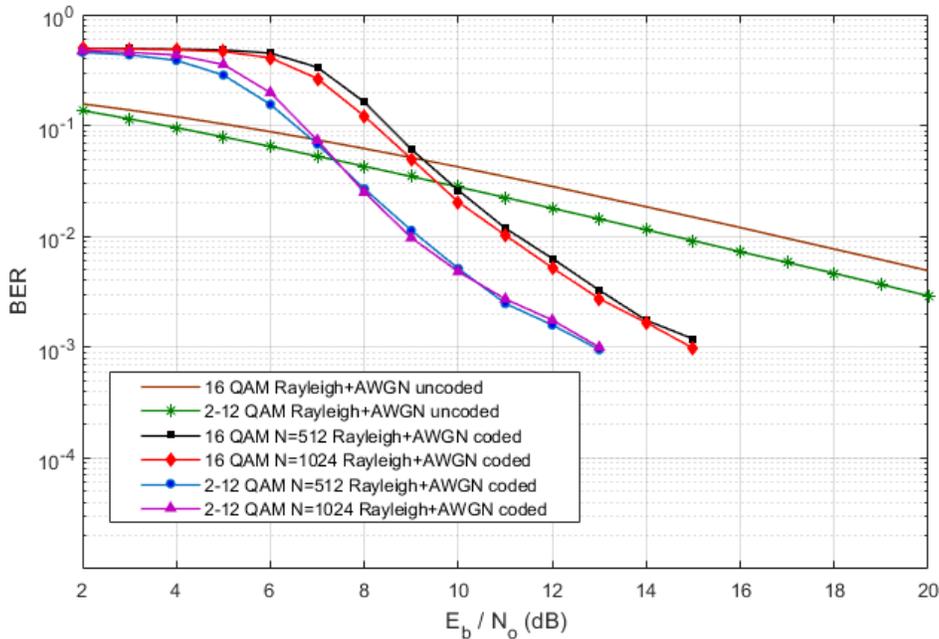
Fig. 7. BER versus E_b/N_0 performance over Rayleigh fading channels disturbed by AWGN of uncoded 2-12-QAM and uncoded 16-QAM, and 2-12-QAM and 16-QAM combined with polar codes, for $N = 512$ and 1024.

1024 and rate $R = 3/4$ in AWGN channel with variance $\sigma^2 \approx 1.67/10^{E_b/N_0}$. The corresponding BER versus E_b/N_0 curve is shown in Fig. 6. The system employing 2-12-QAM modulation and a polar code with block length $N = 1024$ presents a gain of approximately 1.3 dB for BER of 10^{-3}.

Fig. 6 also shows performance curves when a polar code of block length $N = 512$ is employed. When considering 2-12-QAM modulation, the suffix to be added has length $s = 6$ bits and the length of codeword x' is $N_s = 518$. Considering the same spectral efficiency, performance curves for the system using 2-12-QAM with code rate $R = 6/7$ presents an energy gain of approximately 0.8 dB for BER of 10^{-3} when compared to a system using 16-QAM with code rate $R = 3/4$, and block length $N = 512$. The 2-12-QAM combined with a polar code with block length $N = 1024$ presents an energy gain of about 0.6 dB for BER of 10^{-4} when compared to the system that uses 2-12-QAM and a polar code with block length $N = 512$. The system using uncoded 2-12-QAM has an energy gain of about 1.2 dB for BER equal to 10^{-2} when compared with the 16-QAM combined and a polar code with block length $N = 1024$.

Fig. 7 shows performance curves for combined polar codes and 2-12-QAM system, indicating an energy gain of approximately 2 dB for BER equal to 10^{-2} when compared to the system using 16-QAM modulation format with Gray code mapping, both over uncoded AWGN channels and Rayleigh fading channels disturbed by AWGN.

Fig. 7 illustrates the performance of a system using polar codes of rate $R = 6/7$, block length $N = 1024$ with an appended suffix of $s = 5$ bits to achieve compatibility with the 2-12-QAM scheme, and considering AWGN channels and Rayleigh fading channels disturbed by AWGN, when the SC

decoding algorithm is used. This system presents an energy gain of about 2 dB for BER of 10^{-3} over the 16-QAM modulation with Gray code mapping with polar code of rate $R = 3/4$ and block length $N = 1024$ also over AWGN channels and Rayleigh fading channels disturbed by AWGN.

The system performance when 2-12-QAM and a polar code with block length $N = 1024$ are employed, shows an energy gain of about 5.5 dB for BER of 10^{-2} when compared to uncoded 2-12-QAM, both over AWGN channels and Rayleigh fading channels disturbed by AWGN.

The performance of 16-QAM combined with a polar code of block length $N = 1024$ over AWGN channels offers an energy gain of about 6 dB for BER of 10^{-2} in comparison to uncoded 16-QAM over the same channels conditions. For Rayleigh fading channels disturbed by AWGN this system offers an energy gain of about 6 dB for BER equal to 10^{-2} in comparison to uncoded 16-QAM over the same channels conditions. Fig. 7 also shows the performance of systems using 2-12-QAM and 16-QAM both combined with a polar code of block length $N = 512$ and over AWGN channels and Rayleigh fading channels disturbed by AWGN.

VI. CONCLUSIONS

This article presents performance results for a new scheme for data transmission over channels disturbed by AWGN or by Rayleigh fading plus AWGN, using 2-12-QAM modulation and polar codes. The main results are summarized as follows.

- AWGN channels, for a BER of 10^{-4}
  a) Uncoded 2-12-QAM has a 3 dB gain in E_b/N_0 over uncoded 16-QAM.

b) Coded 16-QAM has a 4 dB gain in E_b/N_0 over uncoded 16-QAM, for the same spectral efficiency, employing a polar code of block length 1024.

c) Coded 2-12-QAM has a 1.5 dB gain in E_b/N_0 over coded 16-QAM, for the same spectral efficiency, employing polar codes of block length 1024 in both cases.

- AWGN plus Rayleigh fading, for a BER of 10^{-3}

a) Uncoded 2-12-QAM has a gain of 2.5 dB in E_b/N_0 over uncoded 16-QAM.

b) Coded 16-QAM has a gain of at least 10 dB in E_b/N_0 over uncoded 16-QAM, for the same spectral efficiency, employing a polar code of block length 1024.

c) Coded 2-12-QAM has a 2 dB gain in E_b/N_0 over coded 16-QAM, for the same spectral efficiency, employing either polar codes of block length 1024 or 512.

For future work, we suggest the study of other decoding schemes such as Successive Cancelation List decoding can be used [32] as well as generalized fading channel distributions [33], [34]. Furthermore, we also conjecture that overall system performance might be improved if some redundant bits were used to replace the suffix of zeros appended to each codeword. Finally, as LPDC codes are also in the 3GPP 5G standard, their evaluation in combination with 2-12-QAM can be an interesting research topic.

REFERENCES

[1] M. Ayat, S. Mirzakuchaki, and A. Beheshti-Shirazi, "Design and implementation of high throughput, robust, parallel M-QAM demodulator in digital communication receivers," *IEEE Trans. on Circuits and Syst. I: Reg. Papers*, vol. 63, no. 8, pp. 1295–1304, 2016. doi: 10.1109/TCSI.2016.2589078.

[2] J. Zöllner and N. Loghin, "Optimization of high-order non-uniform QAM constellations," in *2013 IEEE International Symposium on Broadband Multimedia Systems and Broadcasting (BMSB)*. IEEE, 2013, pp. 1–6. doi: 10.1109/BMSB.2013.6621711.

[3] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379–423, 1948. doi: 10.1002/j.1538-7305.1948.tb01338.x.

[4] R. T. Kamurthi, S. R. Chopra, and A. Gupta, "Higher order QAM schemes in 5G UFMC system," in *2020 International Conference on Emerging Smart Computing and Informatics (ESCI)*. IEEE, 2020, pp. 198–202. doi: 10.1109/ESCI48226.2020.9167619.

[5] A. D. Ellis, J. Zhao, and D. Cotter, "Approaching the non-linear Shannon limit," *J. Lightw. Technol.*, vol. 28, no. 4, pp. 423–433, 2009. doi: 10.1109/JLT.2009.2030693.

[6] I. Teikari *et al.*, *Digital predistortion linearization methods for RF power amplifiers*. Finland: Teknillinen korkeakoulu, 2008.

[7] S. Ali, G. Markarian, and E. Arikan, "Novel predistortion algorithm for OFDMA," in *The IEEE 69th Conference on Vehicular Technology (VTC Spring 2009)*, 2009, pp. 1–5. doi: 10.1109/VETECS.2009.5073716.

[8] M. L. M. G. Alcoforado, G. Markarian, and V. C. da Rocha Jr, "Novel 2-12 QAM modulation format," *Proceedings of the O S Popov ONAT*, vol. 17, no. 1, pp. 159–168, 2019.

[9] S. Lin and D. J. Costello, "Error Control Coding," 2004.

[10] Z. B. K. Egilmez, L. Xiang, R. G. Maunder, and L. Hanzo, "The development, operation and performance of the 5G polar codes," *IEEE Commun. Surveys Tuts.*, vol. 22, no. 1, pp. 96–122, 2019. doi: 10.1109/COMST.2019.2960746.

[11] V. Bioglio, C. Condo, and I. Land, "Design of polar codes in 5G new radio," *IEEE Commun. Surveys Tuts.*, 2020. doi: 10.1109/COMST.2020.2967127.

[12] E. Arikan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051–3073, 2009. doi: 10.1109/TIT.2009.2021379.

[13] F. Gabry, V. Bioglio, I. Land, and J.-C. Belfiore, "Multi-kernel construction of polar codes," in *2017 IEEE International Conference on Communications Workshops (ICC Workshops)*. IEEE, 2017, pp. 761–765. doi: 10.1109/ICCW.2017.7962750.

[14] K. Chen, K. Niu, and J.-R. Lin, "An efficient design of bit-interleaved polar coded modulation," in *2013 IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2013, pp. 693–697. doi: 10.1109/PIMRC.2013.6666225.

[15] J. He, K. Wu, Z. Zhou, J. Ma, and J. Shi, "An efficient encoder-subcarrier mapping method combined with polar code for visible light communication," *IEEE Access*, vol. 7, pp. 69 119–69 125, 2019. doi: 10.1109/ACCESS.2019.2916005.

[16] K. Mahender, T. A. Kumar, and K. Ramesh, "Analysis of multipath channel fading techniques in wireless communication systems," *AIP Conference Proceedings*, vol. 1952, no. 1, p. 020050, 2018. doi: 10.1063/1.5032012.

[17] P. K. Deekshith and K. R. Sahasranand, "Polar codes over fading channels with power and delay constraints," *CoRR*, vol. abs/1704.00663, 2017. doi: 10.1109/ISWCS.2017.8108142. [Online]. Available: http://arxiv.org/abs/1704.00663

[18] C. R. Neslen, "Negative conductance load modulation RF power amplifier," Master Dissertation, California Polytechnic State University at San Luis Obispo, California, 2010.

[19] G. M. Drury, G. Markarian, and K. Pickavance, *Coding and Modulation for Digital Television*. Boston, USA: Springer Science & Business Media, 2006, vol. 17.

[20] A. R. Belabad, S. A. Motamedi, and S. Sharifian, "An adaptive digital predistortion for compensating nonlinear distortions in RF power amplifier with memory effects," *Integration*, vol. 57, pp. 184–191, 2017. doi: 10.1016/j.vlsi.2017.01.001.

[21] C. Nader, P. N. Landin, W. Van Moer, N. Bjorsell, and P. Handel, "Performance evaluation of peak-to-average power ratio reduction and digital pre-distortion for OFDM based systems," *IEEE Trans. Microw. Theory and Techn.*, vol. 59, no. 12, pp. 3504–3511, 2011. doi: 10.1109/TMTT.2011.2170583.

[22] M. Salehi and J. Proakis, "Digital Communications," *McGraw-Hill Education*, vol. 31, p. 32, 2007.

[23] S. Le Goff, A. Glavieux, and C. Berrou, "Turbo-codes and high spectral efficiency modulation," in *Proceedings of ICC/SUPERCOMM'94-1994 International Conference on Communications*. IEEE, 1994, pp. 645–649. doi: 10.1109/ICC.1994.368804.

[24] E. Johansson, "List decoding of polar codes," Master Thesis, Lund University, Sweden, October 2017.

[25] R. Mori and T. Tanaka, "Performance of polar codes with the construction using density evolution," *IEEE Commun. Lett.*, vol. 13, no. 7, pp. 519–521, July 2009. doi: 10.1109/LCOMM.2009.090428.

[26] R. M. Oliveira and R. C. de Lamare, "Polar codes based on piecewise Guassian approximation: Design and analysis," *IEEE Access*, vol. 10, pp. 73 571–73 582, July 2022. doi: 10.1109/ACCESS.2022.3190393.

[27] M.-C. Chiu and Y.-S. Su, "Design of polar codes and PAC codes for SCL decoding," *IEEE Trans. Commun.*, vol. 71, no. 3, pp. 2587–2601, March 2023. doi: 10.1109/TCOMM.2023.3263225.

[28] H. Ochiai, K. Ikeya, and P. Mitran, "A new polar code design based on reciprocal channel approximation," *IEEE Trans. Commun.*, vol. 71, no. 2, pp. 631–643, Feb 2023. doi: 10.1109/TCOMM.2022.3227280.

[29] D. Zhou, K. Niu, and C. Dong, "Construction of polar codes in Rayleigh fading channel," *IEEE Commun. Lett.*, vol. 23, no. 3, pp. 402–405, 2019. doi: 10.1109/LCOMM.2019.2892453.

[30] W. T. A. Lopes, Y. Xue, and E. S. Sousa, "Performance of modulation diversity with polar encoding in Rayleigh fading channel," in *Proceedings of the 16th ACM International Symposium on Performance Evaluation of Wireless Ad Hoc, Sensor, & Ubiquitous Networks*, 2019, pp. 51–55. doi: 10.1145/3345860.3361514.

[31] A. A. Hasan and I. D. Marsland, "Low complexity LLR metrics for polar coded QAM," in *2017 IEEE 30th Canadian Conference on Electrical and Computer Engineering (CCECE)*. IEEE, 2017, pp. 1–4. doi: 10.1109/CCECE.2017.7946778.

[32] I. Tal and A. Vardy, "List decoding of polar codes," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2213–2226, 2015. doi: 10.1109/TIT.2015.2410251.

[33] F. C. P. Barros, F. C. M. Oliveira, M. L. M. G. Alcoforado, and W. T. A. Lopes, "Performance of polar codes over generalized fading channels," *Journal of Communication and Information Systems*, vol. 37, no. 1, pp. 1–9, February 2022. doi: 10.14209/jcis.2022.1.

[34] C. R. N. Da Silva, N. Simmons, E. J. Leonardo, S. L. Cotton, and M. D. Yacoub, "Ratio of two envelopes taken from α-μ, η-μ, and κ-

μ variates and some practical applications," *IEEE Access*, vol. 7, pp. 54 449–54 463, 2019. doi: 10.1109/ACCESS.2019.2907891.

Flávia Camila Morais de Oliveira was born in Recife, Pernambuco, Brazil on December 7th, 1995. She received the B.Sc. degrees in Electrical Engineering from the State University of Pernambuco, Brazil, in 2018, where she also received her M.Sc. degree in Systems Engineering in 2021. Her research interests include polar codes, modulation, coding theory and digital signal processing.

Maria de Lourdes Melo Guedes Alcoforado received her B.Sc. degree in electrical and electronics Engineering (1995), M.Sc. (2000) and D.Sc. degrees (2005) in Electrical Engineering from the Federal University of Pernambuco, Brazil. She was a researcher (2013-2016) in the project entitled Coding and Security Advanced Techniques for Wideband Wireless Communications, involving the School of Computing and Communications, Lancaster University and the Communications Research Group, Federal University of Pernambuco. Since 2001 she is with the University of Pernambuco, dedicated to researching and teaching in Undergraduate and Post Graduate courses. She was the Coordinator of the Post Graduate Program in Systems Engineering (2016-2021) and currently she is a Sector Coordinator for Post Graduate Studies and Research at the Polytechnic School of Pernambuco. Her research interests include polar codes, turbo codes, multiple access channel, modulation and cryptography.

Garik Markarian is Emeritus Professor at Lancaster University, UK and the CEO and Founder of Rinicom Ltd (www.rinicom.com). Prior to joining Lancaster University, he spent over ten years in wireless broadband industry, working in senior executive positions for the UK, USA and Canadian technology Companies. He is one of the founding members of the IEEE802.16 standard, where he chaired a number of working groups and actively contributed to the EUROCAE WG-82 (AeroMAX), EUROCAE WG-72 and WG-115 standards. Current research interests include Aviation Security, Secure Communication Systems and Artificial Intelligence for Security Applications. He has co-authored over 700 publications, including 42 patents and 5 text books (including "Aviation Security Engineering: A Holistic Approach" and "Counter-UAV Handbook" both with ARTECH House). He is a Fellow of the National Academy of Sciences in Armenia and for a long time served as the Chair of the IET Professional Panel in Communications. Under his management Rinicom grew up to become one of the world leading technology companies supplying its products to first responders and security agencies around the world. In recognition of these achievements, Rinicom was awarded the Queen's Awards for Excellence in Business in 2013 and 2018.

Valdemar Cardoso da Rocha Jr. was born in Jaboatão, Pernambuco, Brazil, on August 27, 1947. He received his BSc degree in electrical and electronics engineering from Escola Politécnica, Recife, Brazil, in 1970, and his PhD degree in electronics from the University of Kent, England, in 1976. In 1976, he joined the faculty of the Federal University of Pernambuco, Recife, Brazil, founded the Electrical Engineering Postgraduate Program, and since 1993 is Professor of Telecommunications. He has been a consultant to the Brazilian Ministry of Education, the Ministry of Science and Technology, and the Ministry of Defense on postgraduate education and research in electrical engineering. He was Chairman of the Electrical Engineering Committee in the Brazilian National Council for Scientific and Technological Development for two terms. He is a founding member, former President, and Emeritus Member of the Brazilian Telecommunications Society. He is also a Life Senior Member of the IEEE Communications Society and the IEEE Information Theory Society. He is a former Member of the IEEE Alexander Graham Bell Medal Committee and from 1992 to 2022 he was a Fellow of the Institute of Mathematics and its Applications. He has been a Visiting Professor at various institutions, including the Swiss Federal Institute of Technology-Zurich, the University of Leeds and Lancaster University. He has published over 100 engineering and scientific papers, including journal and conference papers, and the books Elements of Algebraic Coding Systems, Momentum Press, 2014, Principles of Applied Digital Information Theory (in Portuguese), Interciencia, 2018, and is a co-author of the book Communication Systems (Springer, 2022, 3rd Edition).

Waslon Terllizzie Araújo Lopes was born in Petrolina, Pernambuco, Brazil on December, 29th, 1974. He received the B.Sc. and M.Sc. degrees in Electrical Engineering from the Federal University of Paraíba, Brazil, in 1998 and 1999, respectively. He received his D.Sc. degree in Electrical Engineering from the Federal University of Campina Grande, Brazil in 2003. From August 2003 to December 2009 he was with ÁREA1 College of Science and Technology in Salvador, Brazil, where he was the head of the Telecommunications Group. From November 2018 to October 2019 he was visiting professor in the University of Toronto, Canada. Currently, Dr. Waslon is Associate Professor in the Department of Electrical Engineering of the Center for Alternative and Renewable Energy of Federal University of Paraíba, Brazil. He is also Financial Director of the Institute for Advanced Studies in Communications (Iecom). His research interests includes robust vector quantization, wireless communication systems, communication theory, and digital signal processing. Dr. Waslon is co-author the books "Communications, Information and Network Security" published by Kluwer Academic Publishers and "Spectral Sensing: Techniques and Applications" published by Momentum Press. Waslon Terllizzie is member of the Brazilian Telecommunications Society (SBrT) since 1997 and Senior Member of the Institute of Electrical and Electronics Engineers (IEEE).