Index Encoding and Antenna Selection in Multiuser Precoder Index Modulation MIMO Communication

Azucena Duarte, João Cal-Braz, and Raimundo Sampaio-Neto

***Abstract*—Index modulation (IM) offers energy efficient solutions to communication systems by altering the on/off status of entities of the system. This work presents a multiuser (MU) IM-based system operating in a multiple-input multiple-output (MIMO) channel, named Multiuser Precoder Index Modulation (MU-PIM-MIMO), in which the choice of the IM-precoder matrices, responsible for assigning zero or nonzero values to the information vector, is a source of information. System model is specified for Zero-Forcing and Block Diagonalization channel precoders, as well as additional mechanisms, such as user notification and channel estimation. Numerical results show that MU-PIM-MIMO systems can offer attractive tradeoff between detection performance and spectral efficiency. Metrics for selection of the most favorable information bearing positions (IBP) patterns of the information vector, based on the maximization of the signal-to-noise ratio and on the maximization of the achievable rate, are developed in order to offer further improvements in system performance. Additionally, a scenario where the number of transmit antenna elements exceeds the number of radiofrequency chains at the base station is considered, and optimal and computational efficient ways to select the IBP patterns and the active transmit antennas are proposed. Simulation results evidence the effectiveness of the strategies.**

***Index Terms*—Multiple-input multiple-output (MIMO); multiuser MIMO; index modulation; precoder index modulation; antenna selection; zero forcing; block diagonalization**

Azucena Duarte is with Tigo Telefónica, Tegucigalpa, Honduras. E-mail: azucenaduarte23@gmail.com.

João Cal-Braz is with the National Institute of Metrology, Quality and Technology, Duque de Caxias, Brazil. E-mail: jabraz@inmetro.gov.br.

Raimundo Sampaio-Neto is with the Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil. E-mail: raimundo@cetuc.puc-rio.br.

Digital Object Identifier: 10.14209/jcis.2022.20

I. INTRODUCTION

The search for new manners of transmitting data with high energy and spectral efficiencies have been concentrating much efforts of scientific community. Index modulation (IM) systems unite strategies that uses the modification of the status of the transmission entities of the system as a source of information. These elements may be, for example, the transmit or receive antennas, transmit light emitting diodes, subcarriers. The energy that is saved from the transmit entities that are inactive is transferred to the active ones, thus leading to communication efficiency. Due to this characteristic, studies have shown improvements in error performance compared to conventional schemes [1].

Embedding the IM strategy to the downlink communication of MU-MIMO systems have been the aim of several research papers lately. These strategies mostly fall within either of the two categories: 1- the ones that apply IM methods to the transmit space [2], [3]. Here, the number of bits associated to the IM is proportional to the ratio of the number of transmit antennas to the number of users. 2- the ones that apply IM to the receive space [4]–[6]. In this case, the number of bits associated to the IM increases as more receive antennas are available at the users. While the former imposes limitations to the number of users that can take part of the system, the latter is bounded by the physical space to accommodate antennas at the users.

Relevant works over the receive-space IM in the downlink of a multiuser MIMO system include [7] which introduces IM on the receive antennas of the users. The research in

[8] proposes enhancements on this system, considering that each user receives multiple parallel streams of data, and the transmitter uses a two-stage preocoder based on zero-forcing and block diagonalization to provide improved multiuser interference suppression. In [4] a similar scenario is considered, but develops a system which partitions the data symbol into broadcast information and information conveyed to individual users. Performance analysis presented in [9] demonstrates that the diversity order of the downlink of receive-space IM MIMO systems is the same of that of conventional MIMO broadcast channel and the coding gain is superior than the conventional system.

This paper is based on a receive-space IM system in the downlink of a multiuser MIMO scenario, here called Precoder Index Modulation (MU-PIM-MIMO). The signal and system model presented herein, based on the initial PIM system developed to single-user scenario [10], uses the precoder matrix as the entity used to encode index information, chosen from a finite set and selected according to the binary data source.

The present paper delves into the study of the influence of the choice of the set of patterns of data symbols applied to the user received information vector, in this paper represented by the "information bearing position (IBP) encoding matrix", on the system performance. Methods to choose the most adequate IBP encoding matrix for the system configuration employed and current wireless channel condition, are developed for Zero-Forcing (ZF) and Block Diagonalization (BD) precoders. In the case of ZF precoding, signal-to-noise maximization corresponds to the optimal selection in the sense of BER minimization. To the best of the authors' knowledge, there are no other works that have performed this kind of study over IM MIMO systems.

Antenna selection in IM MIMO systems refer to the scenario where more antennas are available at the transmitter than the number of transmitter RF chains. Studies of the application of antenna selection in IM MIMO are very scarce. In [11], antenna selection algorithms based on Euclidean distance maximization are employed to select transmit antennas in a spatial modulation single-user system. This paper also presents antenna selection methods in this MU-PIM-MIMO system. In contrast, the work presented in this paper embeds the antenna selection in the IBP encoding matrix optimization problem. As a result, optimal and computational efficient suboptimal algorithms are proposed to select the transmit antenna subset and IBP encoding matrices, for both ZF and BD precoders, and achieve further performance improvement.

Notation: $\binom{a}{b}$ is the binomial coefficient with parameters a, b. $\lfloor \cdot \rfloor$ is the floor operation. $E[\cdot]$ is the statistical expectation operator with respect to a random variable. $\mathrm{Tr}(\mathbf{A})$ and $\mathcal{D}(\mathbf{A})$ are the trace and the diagonal matrix that takes the elements of matrix $\mathbf{A}$ on its nonzero entries, respectively. $\|\mathbf{a}\|$ and $\mathrm{supp}(\mathbf{a})$ are, respectively, the Euclidean norm and the support set of vector $\mathbf{a}$. $|\mathcal{A}|$ represents the cardinality of set $\mathcal{A}$.

II. Signals and Systems Modeling and Preliminaries

Let N_t be the number of base station antennas of a K-user MU-MIMO system, during downlink communication, where each user has N_r receive antennas, and $N_t \geq KN_r$. The string b^k, comprised of binary elements, represents the information to be transmitted from the BS to user k within a slot of time. It contains R_k bits and is mapped into the $N_r \times 1$ information vector $\mathbf{s}^k$. The first k_{pos} bits of b^k in the considered PIM system are encoded for transmission by the selection of an $N_r \times N_r$ precoder matrix, $\mathcal{D}(\mathbf{q}^k)$, out of C_t possible matrices. Vector $\mathbf{q}^k \in \{0, 1\}^{N_r \times 1}$ is such that

$$|\mathrm{supp}(\mathbf{q}^k)| = N_{ibp}. \quad (1)$$

In this IM scheme, determining the information bearing positions (IBP) of the information vector corresponds to choosing the precoding matrix that selects the entries of $\mathbf{s}^k$ that contains non-null elements, determined by $\mathbf{q}^k$. The remaining part of the string b^k corresponds to the transmission of M-ary modulated symbols in the elements of $\mathbf{s}^k$ that contain non-null entries. This section of b^k comprises k_{mod} =

$N_{ibp}\log_2(M)$ bits. Considering that $C_t = \binom{N_r}{N_{ibp}}$ is the number of possible allocations of N_{ibp} non-null elements in $\mathbf{s}^k$, and $k_{pos} = \lfloor \log_2(C_t) \rfloor$, thus $M_{pos} = 2^{k_{pos}}$ is the effective number of IBP patterns that will be used by the BS. Thus, during a timeslot, the number of bits transmitted to the kth user is $R_k = (k_{pos}+k_{mod})$, and $R = \sum_{k=1}^{K} R_k$ is the number of bits transmitted to all users.

In (2) it is shown a possible choice of IBP encoding matrix $\mathbf{Q}^k$, of size $N_r \times M_{pos}$ for a numerical example with $N_r = 4$ and $N_{ibp} = 2$.

$$\mathbf{Q}^k = \begin{bmatrix} \mathbf{q}_1^k & \mathbf{q}_2^k & \mathbf{q}_3^k & \mathbf{q}_4^k \end{bmatrix} = \overset{\text{binary coding}}{\begin{bmatrix} \mathbf{01} & \mathbf{00} & \mathbf{11} & \mathbf{10} \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}. \tag{2}$$

According to the $N_r \times 1$ vector $\mathbf{q}_1^k$ in (2), the information symbols are assigned to the first two entries of $\mathbf{s}^k$ during one transmission. Notice that, as $M_{pos} \leq C_t$, the system selects an IBP encoding matrix $\mathbf{Q}^k$ to be used for communication with user k, out of $L = \binom{C_t}{M_{pos}}$ possible choices. All these possible matrices are gathered in the set $\mathcal{Q} = \{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_L\}$. Each matrix $\mathbf{Q}_l \in \mathcal{Q}$ has a corresponding $N_R \times 1$ mean vector $\overline{\mathbf{q}}(l)$, calculated as

$$\overline{\mathbf{q}}(l) = \frac{1}{M_{pos}} \sum_{i=1}^{M_{pos}} \mathbf{q}_i^k; \quad k = 1, 2, \ldots, K. \tag{3}$$

The ith element of $\overline{\mathbf{q}}(l)$, $i = 1, \ldots, N_R$, is calculated as the mean value along the ith row of $\mathbf{Q}^k$. Taking as an example, $\overline{\mathbf{q}}(l) = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}^T$ corresponds to the $\mathbf{Q}^k$ shown in (2). All L $\overline{\mathbf{q}}(l)$ vectors are organized by the set $= \{\overline{\mathbf{q}}(1), \overline{\mathbf{q}}(2), \ldots, \overline{\mathbf{q}}(L)\}$. In the considered example, $L = 15$ choices. Throughout this work, a deeper view of the impact of the adequate choice of $\mathbf{Q}^k; k = 1, 2, \ldots, K;$ on the communication performance, as well as selection strategies, will be presented.

A. Signal Model

Consider that $\mathbf{s} \in \mathbb{C}^{KN_r \times 1}$ stacks the information vectors transmitted to all K users, that is $\mathbf{s} = \begin{bmatrix} \mathbf{s}^{1^T} & \mathbf{s}^{2^T} & \ldots & \mathbf{s}^{K^T} \end{bmatrix}^T$, and that the information addressed to the kth user is represented by $\mathbf{s}^k; k = 1, 2, \ldots, K$. The positioning vectors in $\mathbf{Q}^k$ determine the non-null entries of $\mathbf{s}^k$, so that $|\mathrm{supp}(\mathbf{q})| = |\mathrm{supp}(\mathbf{s}^k)| = N_{ibp}$. In turn, the non-null entries of $\mathbf{s}^k$ are filled with i.i.d. complex symbols drawn from an M-ary constellation $\mathcal{C}$. The information vectors from different users are assumed statistically independent. Information vectors $\mathbf{s}^k$ are expressed as

$$\mathbf{s}^k = \sqrt{E_k}\,\mathbf{D}(\mathbf{q}^k)\mathbf{c}^k, \tag{4}$$

where the information symbol energy to kth user is E_k, and $\mathbf{q}^k$ is equiprobably drawn from the columns of $\mathbf{Q}^k$, since the k_{pos} bits of b^k, associated to the PIM information, are uniformly distributed. Moreover, $\mathbf{c}^k \in \mathcal{C}^{N_r \times 1}$, with $\mathbb{E}[\mathbf{c}^k] = \mathbf{0}$ and $\mathbb{E}\left[\mathbf{c}^k\mathbf{c}^{k^H}\right] = \mathbf{I}_{N_r}$. Vectors $\mathbf{q}^k$ and $\mathbf{c}^k$ are statistically independent from each other. The N_{ibp} non-null elements of $\mathbf{c}^k$, indicated by $\mathbf{q}^k$, are drawn from $\mathcal{C}$ according to the last k_{mod} elements of b^k, whereas the remaining $(N_r - N_{ibp})$ are arbitrary elements from $\mathcal{C}$ and work as dummy symbols.

The availability of channel state information at the transmitter is assumed. The mth-user channel precoder matrix $\mathbf{P}^m \in \mathbb{C}^{N_t \times N_r}; m = 1, 2, \ldots, K$ is then applied to the index-modulated information vector and aims to separate the users at receiver side. The resulting vector transmitted by the BS antennas is given by

$$\begin{aligned} \mathbf{x} &= \sum_{m=1}^{K} \mathbf{P}^m \mathbf{s}^m \\ &= \sum_{m=1}^{K} \sqrt{E_m}\,\mathbf{P}^m \mathbf{D}(\mathbf{q}^m)\mathbf{c}^m. \end{aligned} \tag{5}$$

B. Energy Relations

Consider that the symbols addressed to the users have average energy $E_s = 1/K \sum_{m=1}^{K} E_m$ and the energy ratio to user m is $\varepsilon_m = E_m/E_s$. By (5), the relation between the average energy of the signal transmitted by the BS, $E_T =$

$E[\|x\|^2]$, and the symbol energy assigned to user k is

$$E_k = E_s \varepsilon_k = E_T \frac{\varepsilon_k}{\gamma}; \quad (6)$$

where

$$\gamma = \sum_{m=1}^{K} \varepsilon_m g_m^T \bar{q}^m; \quad (7)$$

and $\bar{q}^m$ is the IBP encoding matrix Q^m mean vector to be used for transmission to user m, which is calculated as:

$$\bar{q}^m = E[q^m] = \frac{1}{M_{pos}} \sum_{i=1}^{M_{pos}} q_i^m; \quad m = 1, 2, \ldots, K; \quad (8)$$

and

$$\begin{aligned} g_m &= d\left(P^{mH} P^m\right) \\ &= \left[\|p_1^m\|^2 \; \|p_2^m\|^2 \; \ldots \; \|p_{N_r}^m\|^2\right]^T; \\ m &= 1, 2, \ldots, K; \end{aligned} \quad (9)$$

where $d(A)$ is a column vector whose entries are the elements on the main diagonal of any A, and the ith column of P^m, $m = 1, 2, \ldots, K$ is denoted by p_i^m.

Notice that γ acts as a normalization factor applied on the transmitted signal level, which depends on the column norms of the channel precoder matrices of all users and on the IBP encoding matrix mean vector.

Observing (6) and (7), for a given energy E_T at the transmitter, the energy available for the kth user, E_k, is a function of P^m for all K users, via $g_m; m = 1, 2, \ldots, K$, given by (9), and on the IBP encoding matrix, q^m, used for transmission to all users, as shown in (8). A simplified block diagram of the transmitter is shown in Fig. 1.

C. Receivers

The precoded data in (5), transmitted by the BS, can also be written as:

$$x = Ps; \quad (10)$$

where $P \in \mathbb{C}^{N_t \times KN_r}$ is structured as $P = \left[P^1 \; P^2 \; \ldots \; P^K\right]$. Consequently, the K sub-matrices $P^{mH} P^m; m = 1, 2, \ldots, K$ are found centered along the main diagonal of $P^H P$. As a result, the collection of g_m found (9), is expressed as

$$\left[g_1^T \; g_2^T \; \ldots \; g_K^T\right]^T = d\left(P^H P\right). \quad (11)$$

Thus, the signal that traverses the MU-MIMO wireless channels and impinges on the antennas of user k is given by

$$y^k = H_k x + n_k = H_k P s + n_k; \quad (12)$$

here, $H_k \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix that connects the BS antennas to the antennas of kth user, and n_k is recognized as a circularly symmetric complex Gaussian noise, with mean vector and covariance matrix respectively $\mu_{n_k} = 0$ and $K_{n_k} = \sigma_n^2 I_{N_r}$. Thus, the collection of signals received by all users is represented by $y = \left[y^{1T} \; y^{2T} \; \ldots \; y^{KT}\right]^T$ and given by

$$y = Hx + n = HPs + n; \quad (13)$$

where $H = \left[H_1^T \; H_2^T \; \ldots \; H_K^T\right]^T$, and $n = \left[n_1^T \; n_2^T \; \ldots \; n_K^T\right]^T$, then $H \in \mathbb{C}^{KN_r \times N_t}$ and $n \in \mathbb{C}^{KN_r \times 1}$.

In this work, precoders that perfectly decouple signals of different users are employed, meaning that $H_k P^l = 0, k \neq l$. Then

$$y^k = J_k s^k + n_k = \sqrt{E_T \frac{\varepsilon_k}{\gamma}} J_k D\left(q^k\right) c^k + n_k; \quad (14)$$

here $J_k = H_k P^k$ is recognized as the effective channel matrix.

The optimal maximum likelihood detector of the information vector s^k is considered in the strategies presented throughout this work. For detection purposes, this vector can be conveniently represented as $D\left(q^k\right) c^k = A^k b^k$, where all entries of $b^k \in \mathbb{C}^{N_{ibp} \times 1}$ are zero-mean and unit-variance symbols drawn from $\mathcal{C}$, and A^k is formed by the columns of the identity matrix of size $N_r \times N_r$ indicated by the positions of the N_{ibp} unit entries of q^k. As a result, the set of position matrices $\mathcal{A}_k = \left\{A_1^k, A_2^k, \ldots, A_{M_{pos}}^k\right\}$ substitutes the IBP encoding matrix Q^k. The

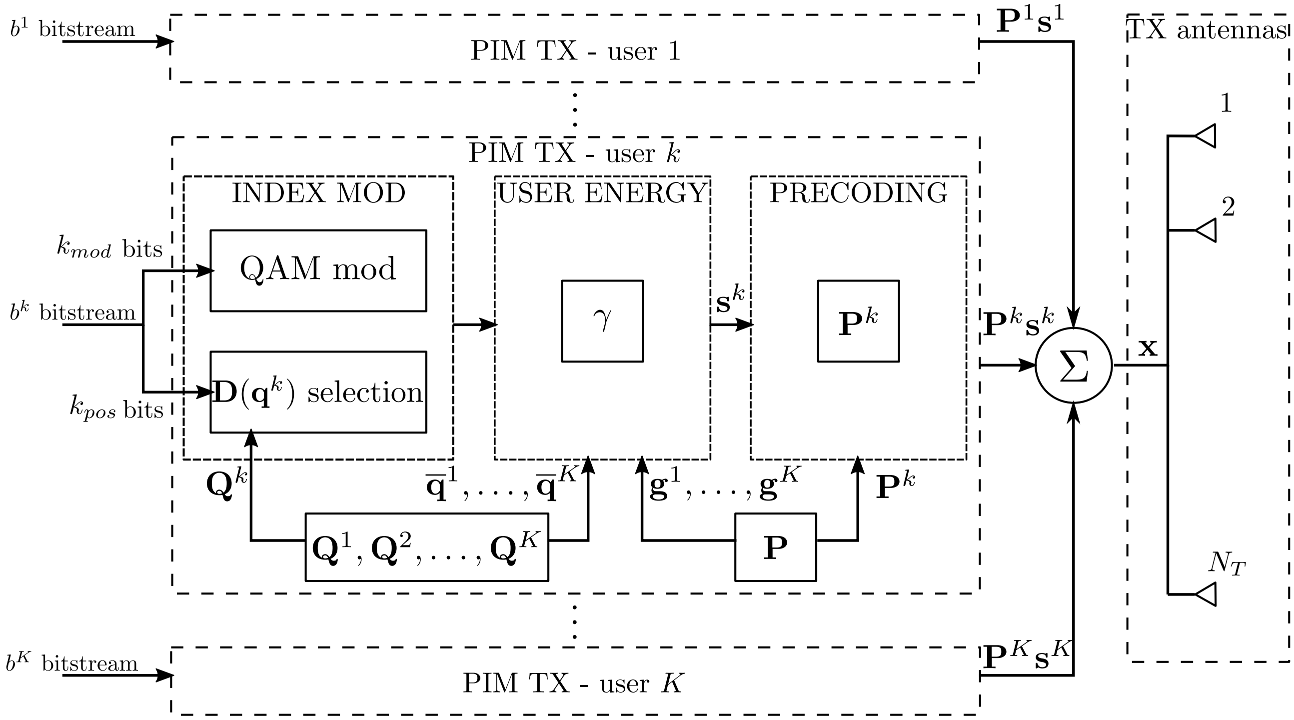


Fig. 1: Block diagram of the MU-PIM-MIMO transmitter.

maximum likelihood detector performs the minimization over the set of position matrices and the modulation set, as given by:

$$(\hat{\mathbf{A}}^k, \hat{\mathbf{b}}^k) = \arg\min_{\substack{\mathbf{A} \in \mathcal{A}_k \\ \mathbf{b} \in \mathcal{C}^{N_{ibp}}}} \left\| \mathbf{y}^k - \sqrt{E_T \frac{\epsilon^k}{}}\, \mathbf{J}_k \mathbf{A} \mathbf{b} \right\|^2 . \quad (15)$$

D. Signal-to-noise ratio metrics

System signal-to-noise ratio referred to the energy conveyed by the transmitter, SNR, and the signal-to-noise ratio per transmitted bit, SNR_{bit}, are given by

$$SNR = \frac{E_T}{\sigma_n^2} \quad (16)$$

$$SNR_{bit} = \frac{SNR}{R} \quad (17)$$

The received SNR per bit, $SNR^k_{r,bit}$ of a user k is defined as the ratio of the mean information signal energy of this user in (14), E^r_k, to the noise variance at the antennas of the user, divided by the number of information bits in a transmission slot. This is expressed as

$$SNR^k_{r,bit} = \frac{1}{R_k} \frac{E^r_k}{\sigma_n^2} \quad (18)$$

where E^r_k, obtained from (14), is expressed as

$$E^r_k = E_T \frac{\epsilon^k}{} \mathrm{Tr}\left(\mathbf{J}_k{}^H \mathbf{J}_k \mathbf{D}(\overline{\mathbf{q}}^k) \right) . \quad (19)$$

Replacing $SNR = R\, SNR_{bit}$ and considering $R_1 = R_2 = \ldots = R_K = R_{user}$, then $R = K R_{user}$ and $SNR^k_{r,bit}$ in (18) is developed as:

$$SNR^k_{r,bit} = \frac{\epsilon^k}{} \mathrm{Tr}\left(\mathbf{J}_k^H \mathbf{J}_k \mathbf{D}(\overline{\mathbf{q}}^k) \right) K\, SNR_{bit} . \quad (20)$$

E. Channel capacity and achievable rate

Multiuser channel capacity is characterized by a rate region, and each point within it represents the vector of achievable rates that can be maintained simultaneously by all users with error probability arbitrarily small. The sum-rate capacity is a boundary point of the capacity region which corresponds to the maximum of the sum of the rates that can be conveyed by the transmitter to the receivers.

Consider a scenario with K non cooperative users in a broadcast multiuser MIMO channel, in which the base station has perfect knowledge of the channel. If the precoder submatrix $\mathbf{P}_k$ is maintained fixed, the kth user achievable rate, I_k, is expressed as:

$$I_k = \log_2 \left[\det \left(\mathbf{K}_{\bar{k}}^{-1} \mathbf{H}_k \mathbf{P}^k \mathbf{K}_{\mathbf{z}_k} \mathbf{P}^{k^H} \mathbf{H}_k^H + \mathbf{I}_{N_r} \right) \right] ; \quad (21)$$

where $\mathbf{K}_{\mathbf{z}_k} = \mathrm{E}[\mathbf{z}_k \mathbf{z}_k^H]$ is the covariance matrix of the kth user data signal at the receiver. The interference plus noise covariance matrix, $\mathbf{K}_{\bar{k}}$, is given by

$$\mathbf{K}_{\bar{k}} = \mathbf{K}_{\mathbf{n}_k} + \sum_{l=1\, l\neq k}^{K} \mathbf{H}_k \mathbf{P}^l \mathbf{K}_{\mathbf{z}_l} \mathbf{P}^{l^H} \mathbf{H}_k^H . \quad (22)$$

Considering a MU-PIM-MIMO system with $\mathbf{z}_k = \sqrt{E_T \frac{\varepsilon}{k}} \mathbf{D}(\mathbf{q}^k)\, \mathbf{c}^k$, $\mathbf{J}_k = \mathbf{H}_k \mathbf{P}^k$ and decoupling precoders, it follows that

$$\mathbf{K}_{\mathbf{z}_k} = E_T \frac{\varepsilon}{k} \mathbf{D}\left(\bar{\mathbf{q}}^k\right) \otimes \mathbf{I}_{N_r} \quad (23)$$

and

$$\mathbf{K}_{\bar{k}}^{-1} = \frac{1}{\sigma_n^2} \mathbf{I}_{N_r} . \quad (24)$$

Then (21) is rewritten as

$$I_k = \log_2 \left[\det \left(SNR \frac{\varepsilon}{k} \mathbf{J}_k \mathbf{D}\left(\bar{\mathbf{q}}^k\right) \mathbf{J}_k^H + \mathbf{I}_{N_r} \right) \right] \quad (25)$$

where SNR is given by (16).

Expression (25) evidences that the kth user achievable rate can be maximized for the proper choice of its parameters, what includes the mean vector of the user IBP encoding matrix, $\bar{\mathbf{q}}^k$. In sections to follow in this work, the influence of the choice of $\mathbf{Q}^k$ on the detection performance will be further investigated.

F. Semi-analytical error probability upper bound

The user symbol vector error detection probability in a multiuser communication system, denoted by $\mathrm{P}_k(\text{error})$, corresponds to the expectation of the error probability conditional to $\mathbf{H}$, given by

$$\mathrm{P}_k(\text{error}) = \mathrm{E}_{\mathbf{H}}[\mathrm{P}_k(\text{error}|\mathbf{H})] \quad (26)$$

In turn, using the union bound of error events, the conditional error probability of user k, $\mathrm{P}_k(\text{error}|\mathbf{H})$ is upper bounded by (27), that is a function of pairwise error probabilities (PEPs):

$$\mathrm{P}_k(\text{error}|\mathbf{H}) = \sum_{\mathbf{s}_i^k} \mathrm{P}\left(\mathbf{s}_i^k\right) \mathrm{P}\left(\bigcup_{\mathbf{s}_j^k;\, \mathbf{s}_j^k \neq \mathbf{s}_i^k} \mathbf{s}_i^k \to \mathbf{s}_j^k \,\Big|\, \mathbf{H} \right) \le \sum_{\mathbf{s}_i^k} \mathrm{P}\left(\mathbf{s}_i^k\right) \sum_{\mathbf{s}_j^k;\, \mathbf{s}_j^k \neq \mathbf{s}_i^k} \mathrm{P}\left(\mathbf{s}_i^k \to \mathbf{s}_j^k | \mathbf{H}\right) . \quad (27)$$

where the PEP $\mathrm{P}\left(\mathbf{s}_i^k \to \mathbf{s}_j^k|\mathbf{H}\right)$ corresponds to the probability of choosing $\mathbf{s}_j$ when $\mathbf{s}_i$ was actually transmitted, given the channel matrix $\mathbf{H}$. Considering a minimum distance detector in presence of additive white Gaussian noise, PEP is given by

$$\mathrm{P}\left(\mathbf{s}_i^k \to \mathbf{s}_j^k|\mathbf{H}\right) = \mathrm{Q}\left(\sqrt{\frac{d_{ij}^2}{2\sigma_n^2}} \right) ; \quad (28)$$

where σ_n^2 is the variance of the components of the noise vector, and d_{ij} is the Euclidean distance between the ith and the jth received information vector, given in (14)

$$d_{ij} = \sqrt{E_T \frac{\varepsilon}{k}}\, d'_{ij} ; \quad (29)$$

with

$$d'_{ij} = \left\| \mathbf{J}_k \left(\mathbf{D}\left(\mathbf{q}_i^k\right) \mathbf{c}_i^k - \mathbf{D}\left(\mathbf{q}_j^k\right) \mathbf{c}_j^k \right) \right\| ; \quad (30)$$

and $\mathrm{Q}(\cdot)$ function corresponds to

$$\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{\alpha^2}{2} \right) d\alpha . \quad (31)$$

Using (29) in (28), and (16), the conditional PEP is written as

$$\mathrm{P}\left(\mathbf{s}_i^k \to \mathbf{s}_j^k|\mathbf{H}\right) = \mathrm{Q}\left(\sqrt{\frac{\varepsilon}{2k} {d'_{ij}}^2 SNR} \right) . \quad (32)$$

Thus, the kth user conditional bit error probability, $P_k^{bit}(\text{error}|\mathbf{H})$, is bounded by

$$P_k^{bit}(\text{error}|\mathbf{H}) \le \sum_{\mathbf{s}_i^k} P\left(\mathbf{s}_i^k\right) \sum_{\substack{\mathbf{s}_j^k;\\ \mathbf{s}_j^k \neq \mathbf{s}_i^k}} \frac{d\left(\mathbf{s}_i^k;\mathbf{s}_j^k\right)}{R_k} P\left(\mathbf{s}_i^k \to \mathbf{s}_j^k|\mathbf{H}\right)$$
$$= \frac{1}{NR_k}\sum_{\mathbf{s}_i^k}\sum_{\substack{\mathbf{s}_j^k;\\ \mathbf{s}_j^k \neq \mathbf{s}_i^k}} d\left(\mathbf{s}_i^k;\mathbf{s}_j^k\right) P\left(\mathbf{s}_i^k \to \mathbf{s}_j^k|\mathbf{H}\right) \quad (33)$$

where $N = 2^{R_k}$ and $d\left(\mathbf{s}_i^k;\mathbf{s}_j^k\right)$ denote the Hamming distance between binary codings of $\mathbf{s}_i^k$ and $\mathbf{s}_j^k$.

In a semi-analytic perspective, an approximation to the upper bound of the pairwise error probability is numerically obtained via Monte Carlo method, as the error rate computed from N_{CR} independent outcomes of the channel matrix random variable $\mathbf{H}$, using (33) and the approximation

$$P_k^{bit}(\text{error}) = E_{\mathbf{H}}[P_k^{bit}(\text{error}|\mathbf{H})] \approx \frac{1}{N_{CR}}\sum_{h=1}^{N_{CR}} P_k^{bit}\left(\text{error}|\mathbf{H}^{(h)}\right); \quad (34)$$

where $\mathbf{H}^{(h)}$ is the hth outcome of the random experiment.

Note that the dependence of the channel $\mathbf{H}$ on the pairwise error probability in (32) occurs by means of and d'_{ij}, where the vectors $\mathbf{g}_m; m = 1; 2 \dots; K$ in (11) are required to the computation of in (7) and d'_{ij} in (30) depends on the effective channel matrix, $\mathbf{J}_k = \mathbf{H}_k\mathbf{P}^k$.

Although the error probability expression presented is not a closed-form solution, the Monte Carlo simulation required for the computation of the upper-bound in (34) is significantly less computational intense compared to bit-error rate computation, when the full system model is considered.

III. Precoders and MU-PIM-MIMO System

A. Zero Forcing Precoding

1) ZF Precoder: The right pseudoinverse of $\mathbf{H}$ is used in the ZF precoder matrix:

$$\mathbf{P}_{ZF} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}; \quad (35)$$

resulting in $\mathbf{J}_k^{ZF} = \mathbf{I}_{N_r}$. Thus, (14) is developed as

$$\mathbf{y}^k = \mathbf{s}^k + \mathbf{n}_k = \sqrt{E_T \tfrac{k}{}}\,\mathbf{D}\left(\mathbf{q}^k\right)\mathbf{c}^k + \mathbf{n}_k. \quad (36)$$

The scalar factor $\sqrt{E_T \tfrac{k}{}}$ turns the received power at user k into a floating value. Although depends on the value of the full channel matrix, in practice its knowledge is not a requirement for detection of the information vector. With the aid of an automatic gain controller, the level of the received signal can be adjusted to prescribed values (as done, for example, in the detection of M-ary QAM, $M > 4$, signals). Alternatively, user k can perform the estimation of the scalar $\sqrt{E_T \tfrac{k}{}}$ prior to detection.

2) IBP encoding matrix selection:

a) User signal-to-noise ratio maximization: The signal-to-noise ratio for the ZF precoding resulting from (20) is given by

$$SNR_{r;bit}^k = \frac{k}{}N_{ibp}K\,SNR_{bit}; \quad (37)$$

once $\text{Tr}\{\mathbf{D}(\overline{\mathbf{q}}^m)\} = N_{ibp}$.

Choosing the IBP encoding matrices $\mathbf{Q}^m; m = 1; 2; \dots; K;$ that leads to the minimization of in (6) and (7) results in the maximization of the $SNR_{r;bit}^k$ in (37), as shown in (38):

$$\mathbf{Q}_{sel;ZF}^1; \mathbf{Q}_{sel;ZF}^2; \dots; \mathbf{Q}_{sel;ZF}^K = \arg\min_{\mathbf{Q}^1;\mathbf{Q}^2;\dots;\mathbf{Q}^K \in \mathcal{Q}} \quad . \quad (38)$$

Since all terms of the summation in (7) are positive and each one is function of the characteristics associated to a single user, the problem of minimizing is reduced to the independent minimization of the terms in (7). Analyzing (11) and (35), the vectors $\mathbf{g}_m, m = 1, 2, \ldots, K,$ in (7) correspond to

$$\begin{aligned} \left[\mathbf{g}_1^T \ \mathbf{g}_2^T \ \ldots \ \mathbf{g}_K^T\right]^T &= \mathrm{d}\left(\mathbf{P}_{ZF}^H \mathbf{P}_{ZF}\right) \\ &= \mathrm{d}\left(\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\right). \end{aligned} \quad (39)$$

Moreover, to each matrix $\mathbf{Q}_l \in \mathcal{Q}$ corresponds a mean vector $\bar{\mathbf{q}}(l) \in$, as in (8). So, optimally choosing the IBP encoding matrix of the kth user, $\mathbf{Q}_{opt}^k$, is equivalent to

$$\mathbf{Q}_{opt}^k = \mathbf{Q}_{l^{ZF}(k)}, \quad (40)$$

where

$$l^{ZF}(k) = \arg\min_{1 \le l \le L} \mathbf{g}_k^T \bar{\mathbf{q}}(l). \quad (41)$$

Commonly, different IBP encoding matrices belonging to $\mathcal{Q}$ exhibit equal $\bar{\mathbf{q}}$. These $\mathbf{Q}_l$ are considered duplicates in the minimization problem and result in the same system performance, thus, can be discarded from $\mathcal{Q}$. This reduces the cardinality of $\mathcal{Q}$ and, consequently, the complexity of the optimization.

b) Achievable rate maximization: The kth user achievable rate is dependent on the choice of the IBP encoding matrices of all users. Considering (25) and (36), $I_k(\bar{\mathbf{q}}^1, \bar{\mathbf{q}}^2, \ldots, \bar{\mathbf{q}}^K)$ is given by

$$I_k(\bar{\mathbf{q}}^1, \bar{\mathbf{q}}^2, \ldots, \bar{\mathbf{q}}^K) = \sum_{i=1}^{N_r} \log_2\left(SNR \frac{\varepsilon_k}{(\bar{\mathbf{q}}^1, \bar{\mathbf{q}}^2, \ldots, \bar{\mathbf{q}}^K)} \bar{q}_i^k + 1\right), \quad (42)$$

where $(\bar{\mathbf{q}}^1, \bar{\mathbf{q}}^2, \ldots, \bar{\mathbf{q}}^K)$, obtained from (7) and (39), makes explicit the dependency of on the choice of the IBP encoding patterns of the users.

The sum-rate of the system is expressed as

$$I_T(\mathcal{L}) = \sum_{m=1}^{K} I_m(\mathcal{L}), \quad (43)$$

where $\mathcal{L} = \{l^1, l^2, \ldots, l^K\}$ is the ordered set containing the indices of the IBP encoding matrices employed by the users.

The possible hypotheses of the IBP encoding matrices indices for the computation of the achievable rate are organized in $\mathbb{L} = \{\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_{L^K}\}$. Thus, the maximum sum-rate is obtained by the choice of the IBP mean vectors $\bar{\mathbf{q}}^1(l^1), \bar{\mathbf{q}}^2(l^2) \ldots, \bar{\mathbf{q}}^K(l^K)$ (and, consequently, by the corresponding IBP encoding matrices) indicated by the set $\mathcal{L}_{esc}$ given by

$$\mathcal{L}_{esc} = \mathcal{L}_{i^*}, \quad (44)$$

where

$$i^* = \arg\max_{i \in \{1, 2, \ldots, L^K\}} I_T(\mathcal{L}_i) \quad (45)$$

with $\mathcal{L}_i \in \mathbb{L}$. Then, the IBP encoding matrix selected for the kth user is given by

$$\mathbf{Q}_{cap,ZF}^k = \mathbf{Q}_{l_c^{ZF}(k)}, \quad (46)$$

where $l_c^{ZF}(k) \in \mathcal{L}_{esc}$.

3) User notification: In the case of ZF-precoded system, the base station needs to inform each user which of the possible L IBP encoding matrices will be used for transmission. This notification scheme is done by means of a frame based transmission. At the end of each frame a new choice of $\mathbf{Q}^m$ is done by the transmitter, followed by the notification of the users.

In the case where $N_r = 4$ and $N_{ibp} = 2$, BS and user may agree in advance to communicate the IBP encoding matrix information emitting nonzero symbols in fixed N_{ibp} entries of the information vector (i.e. no IM used in notification). If QPSK constellation ($M = 4$) is employed, all $L = 15$ encoding matrices indices can be encoded within a channel use. Considering that the same information is sent over F_{notf} times repeatedly, fixed channel condition, and the same additive random noise distribution is observed at the received signal, this results in a processing gain which increases the signal-to-noise ratio by a factor of $10\log_{10}(F_{notf})$ dB [12] resulting in more reliable notification information arriving at the users.

B. Block Diagonalization Precoding

1) BD Precoder: This precoder requires two singular value decompositions (SVD), $\mathbf{P}_a^k$ and

$\mathbf{P}_b^k$, and can be expressed as $\mathbf{P}_{BD}^k = \mathbf{P}_a^k\mathbf{P}_b^k$. The received signal in (12) is rewritten as

$$\mathbf{y}^k = \mathbf{H}_k\mathbf{P}_{BD}^k\mathbf{s}^k + \sum_{l=1;\, l\neq k}^{K} \mathbf{H}_k\mathbf{P}_{BD}^l\mathbf{s}^l + \mathbf{n}_k. \quad (47)$$

Choosing $\mathbf{P}_a^l$ to belong to the null space of $\mathbf{H}_k$ results in the interference cancellation produced by other users signals on the kth user. Denote by $\bar{\mathbf{H}}_k$ the channel matrix that connects the BS to all users, except for the kth user

$$\bar{\mathbf{H}}_k = \left[\mathbf{H}_1^T \;\ldots\; \mathbf{H}_{k-1}^T \;\mathbf{H}_{k+1}^T, \ldots \;\mathbf{H}_K^T\right]^T. \quad (48)$$

Thus, designing $\mathbf{P}_a^k$ such that $\bar{\mathbf{H}}_k\mathbf{P}_a^k = \mathbf{0}_{(K-1)N_r \times N_r}$, inter user interference is removed.

In block diagonalization, SVD is used to obtain $\mathbf{P}_a^k$. The rank of $\bar{\mathbf{H}}_k$ is $r = \min((K-1)N_r, N_t) = (K-1)N_r$ and its SVD is written as

$$\bar{\mathbf{H}}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \left[\mathbf{V}_k^{(1)}\mathbf{V}_k^{(0)}\right]^H. \quad (49)$$

The left singular vector and singular values matrices of $\bar{\mathbf{H}}_k$ are $\mathbf{U}_k \in \mathbb{C}^{(K-1)N_r \times (K-1)N_r}$ and $\boldsymbol{\Sigma}_k \in \mathbb{C}^{(K-1)N_r \times N_t}$ in (49). The right singular vectors associated to the non-null singular values and to the null singular values are $\mathbf{V}_k^{(1)} \in \mathbb{C}^{N_t \times r}$ and $\mathbf{V}_k^{(0)} \in \mathbb{C}^{N_t \times (N_t - r)}$, and are contained in the vector space and in the null space of $\bar{\mathbf{H}}_k$, respectively. Hence, $\mathbf{P}_a^k = \mathbf{V}_k^{(0)}$. Here, the $N_r \times (N_t - r)$ matrix $\mathbf{H}_k^{'}$ is defined as $\mathbf{H}_k^{'} \triangleq \mathbf{H}_k\mathbf{P}_a^k$ (notice $N_t - r = N_t - KN_r + N_r \geq N_r$).

In turn, $\mathbf{H}_k^{'}$ is decomposed into singular vectors to form $\mathbf{P}_b^k$. The SVD of $\mathbf{H}_k^{'}$ is expressed as:

$$\mathbf{H}_k^{'} = \mathbf{U}_k^{'}\boldsymbol{\Sigma}_k^{'}\mathbf{V}_k^{'H}. \quad (50)$$

The first N_r columns of the $(N_t - r) \times (N_t - r)$ matrix $\mathbf{V}_k^{'}$ are used to compose $\mathbf{P}_b^k$.

User k receives the signal that is expressed as

$$\begin{aligned}\mathbf{y}^k &= \mathbf{J}_k^{BD}\mathbf{s}^k + \mathbf{n}_k \\ &= \sqrt{E_T\frac{\varepsilon_k}{\lambda}}\,\mathbf{J}_k^{BD}\mathbf{D}(\mathbf{q}^k)\mathbf{c}^k + \mathbf{n}_k, \end{aligned} \quad (51)$$

where $\mathbf{J}_k^{BD} = \mathbf{U}_k^{'}\boldsymbol{\Sigma}_k^{'}$, $\mathbf{J}_k^{BD} \in \mathbb{C}^{N_r \times N_r}$. The singular values matrix $\boldsymbol{\Sigma}_k^{'}$ contains along its main diagonal the N_r nonzero singular values of $\mathbf{H}_k^{'}$.

2) IBP encoding matrix selection:

a) User signal-to-noise ratio maximization: Observing the relationship between λ and $\mathbf{g}_m$, given in (7), if BD precoding is considered, vectors $\mathbf{g}_m$ are given by

$$\left[\mathbf{g}_1^T \;\mathbf{g}_2^T \;\ldots\; \mathbf{g}_K^T\right]^T = \mathrm{d}\left(\mathbf{P}_{BD}^H\mathbf{P}_{BD}\right) = \mathbf{1}_{KN_r}. \quad (52)$$

Expression (52) yields parameter λ to have the following expression:

$$\begin{aligned}\lambda &= \sum_{m=1}^{K} \varepsilon_m \mathbf{1}_{N_r}^T\bar{\mathbf{q}}^m \\ &= \sum_{m=1}^{K} \varepsilon_m \mathrm{Tr}\{\mathbf{D}(\bar{\mathbf{q}}^m)\} \\ &= KN_{ibp}. \end{aligned} \quad (53)$$

The constant value of λ for any channel realization makes the signal energy directed to the users, E_k in (6), to be independent of the precoder matrix. E_k is then written as

$$E_k = E_T\frac{\varepsilon_k}{\lambda} = E_T\frac{\varepsilon_k}{KN_{ibp}}. \quad (54)$$

As the maximization of the signal-to-noise ratio at the user is adopted as the criterion for IBP encoding matrix selection, from (51) the total signal energy at kth user is

$$E_k^r = E_k\mathbf{v}_k^T\bar{\mathbf{q}}^k, \quad (55)$$

where $\mathbf{v}_k$ is identified as

$$\begin{aligned}\mathbf{v}_k &= \mathrm{d}\left(\mathbf{J}_k^{BD\,H}\mathbf{J}_k^{BD}\right) = \mathrm{d}\left(\boldsymbol{\Sigma}_k^{'2}\right) \\ &= \left[\sigma_1^k \;\sigma_2^k \;\ldots\; \sigma_{N_r}^k\right]^T. \end{aligned} \quad (56)$$

Recalling that SNR_{bit} is defined in (17), then, each user observes a signal-to-user ratio per received bit, $SNR_{r,bit}^k$, that is expressed as

$$SNR_{r,bit}^k = \frac{\varepsilon_k}{N_{ibp}}\mathbf{v}_k^T\bar{\mathbf{q}}^k SNR_{bit}. \quad (57)$$

Maximizing $\mathbf{v}_k^T\bar{\mathbf{q}}^k$ leads to the maximization of $SNR_{r,bit}^k$. Then, the selected IBP encoding matrix, $\mathbf{Q}_{sel}^k$, is determined as

$$\mathbf{Q}_{sel,BD}^k = \mathbf{Q}_{l^{BD}(k)}, \quad (58)$$

where

$$l^{BD}(k) = \arg\max_{1\leq l\leq L} \mathbf{v}_k^T\bar{\mathbf{q}}(l). \quad (59)$$

b) Achievable rate maximization: The kth user achievable rate in BD-precoded system is obtained using the effective channel matrix $\mathbf{J}_k^{BD}$ in (25). Using the result in (53) for , the achievable rate results in

$$I_k = \sum_{i=1}^{N_r} \log_2\left(SNR \frac{\varepsilon_k}{K N_{ibp}} \bar{q}_i^k \lambda_i^k + 1\right); \quad (60)$$

where $\bar{q}_i^k$ is the ith component of vector $\bar{\mathbf{q}}^k$ given by (8) and λ_i^k is given by (56). Clearly, the IBP encoding matrix that maximizes each user achievable rate is selected as

$$\mathbf{Q}_{cap;BD}^k = \mathbf{Q}_{l_c^{BD}(k)}; \quad (61)$$

where

$$l_c^{BD}(k) = \arg\max_{l \in \{1,2,\ldots,L\}} I_k(l^k). \quad (62)$$

Similarly, the selection of the IBP encoding matrix that maximizes each user achievable rate can be additionally simplified if large values of SNR are considered. In this case, I_k is given by

$$I_k^{assin} \approx N_r \log_2\left(SNR \frac{\varepsilon_k}{K N_{ibp}}\right) + \log_2\left[\prod_{i=1}^{N_r} \lambda_i^k\right] + \log_2\left[\prod_{i=1}^{N_r} \bar{q}_i^k\right].$$

The IBP encoding matrix selection is carried out by the maximization of the last term of the summation

$$l_c^{BD}(k) = \arg\max_{l \in \{1,2,\ldots,L\}} \prod_{i=1}^{N_r} \bar{q}_i(l); \quad (63)$$

where $\bar{q}_i(l)$ is the ith entry of $\bar{\mathbf{q}}(l) \in$.

3) Estimation of $\mathbf{J}_k^{BD}$: The estimation of $\mathbf{J}_k^{BD}$ is obtained by the least-squares estimator. F_{est} pilot vectors are transmitted, forming an $N_r \times F_{est}$ matrix $\underline{\mathbf{S}}^k$, where $F_{est} \geq N_r$. $\mathbf{Y}^k$ is received by user k:

$$\mathbf{Y}^k = \mathbf{J}_k^{BD}\underline{\mathbf{S}}^k + \mathbf{N}_k; \quad (64)$$

where $\mathbf{Y}^k = [\mathbf{y}_1^k \; \mathbf{y}_2^k \; \ldots \; \mathbf{y}_{F_{est}}^k]$ is the observation matrix and $\mathbf{N}_k = [\mathbf{n}_k^1 \; \mathbf{n}_k^2 \; \ldots \; \mathbf{n}_k^{F_{est}}]$ is the noise matrix.

If the components of the pilot sequence have equal magnitude and energy E_k^p, and they are chosen to be orthogonal to each other, then $\underline{\mathbf{S}}^k\underline{\mathbf{S}}^{k^H} = F_{est}E_k^p\mathbf{I}_{N_r}$. In this case, the least-squares estimate of $\mathbf{J}_k^{BD}$, $\hat{\mathbf{J}}_k^{BD}$, is given by

$$\hat{\mathbf{J}}_k^{BD} = \mathbf{Y}^k\underline{\mathbf{S}}^{k^H}\left(\underline{\mathbf{S}}^k\underline{\mathbf{S}}^{k^H}\right)^{-1} = \frac{1}{F_{est}E_k^p}\mathbf{Y}^k\underline{\mathbf{S}}^{k^H}. \quad (65)$$

After the estimation of $\mathbf{J}_k^{BD}$ is done by the receiver, it can determine the optimal IBP encoding matrix selection in use by the transmitter in either of two ways: a- following the procedure in (58), (59) and (56) using the effective channel estimate $\hat{\mathbf{J}}_k^{BD}$, or b- the transmitter notifies the users about the selected IBP encoding matrix, using a notification procedure as presented in Sec. III-A3.

IV. Numerical Results - Index Encoding Matrix Selection

Monte Carlo simulations are carried out to assess the performance of ZF- and BD-precoded MU-PIM-MIMO systems. Zero-mean unit-variance circularly symmetric complex Gaussian elements compose the kth user channel matrix $\mathbf{H}_k; k = 1, 2, \ldots, K$. The same amount of energy to all users is assumed ($\varepsilon_k = 1$), as well as QPSK modulation ($M = 4$). The performance curves are generated after $1{,}000$ realizations of $\mathbf{H}$. In each channel realization, each user receives $19{,}200$ bits from the BS.

In the baseline system configuration, no IBP encoding matrix selection is performed. In order to prevent any bias in the results, IBP encoding matrices $\mathbf{Q}^k; k = 1, 2, \ldots, K$ are randomly chosen from the set $\mathcal{Q}$ whenever a new realization of the channel matrix is generated. Throughout this section, this baseline configuration is labeled as $\mathbf{Q}$-Rand in the figures. Performance results are expressed in terms of SNR_{bit}, as given in (17).

In Fig. 2, the performances of ZF- and BD-precoded system are shown for $\{N_t, (N_r, N_{ibp}), K\} = \{8, (4, N_{ibp}), 2\}$, when no IBP encoding matrix selection strategy is employed. The number of nonzero entries of $\mathbf{s}^k$, N_{ibp}, is varied, while other parameters remain unaltered. From the curves, we can conclude that smaller values of N_{ipb} leads to significant performance improvements at some reduction in spectral efficiency, as shown in Table I.

The plots in Fig. 3 exhibit BER performance curves of MU-PIM-MIMO systems employing ZF precoding (Fig. 3a) and BD precoding (Fig. 3b), where IBP-selection-based schemes, presented in Sec. III-A2, III-A3, III-B2 and III-B3, are compared to the same system employing randomly chosen IBP encoding matrix, $\mathbf{Q}$-Rand, for the system configuration: $\{8, (4, 2), 2\}$. Comparing $\mathbf{Q}$-Rand to $\mathbf{Q}$-SNR (based on the maximization of the $SNR^k_{r;bit}$), in which the knowledge of the selected IBP encoding matrix by the users is assumed, for a fixed error rate value, selecting the IBP encoding matrix via $\mathbf{Q}$-SNR provides savings of 1 dB and 2.5 dB of SNR_{bit} for both ZF- and BD-precoded systems.

Particularly for ZF-precoded systems, in Fig. 3a, $\mathbf{Q}$-SNR-Notf (in dashed line) assumes that the $\mathbf{Q}^k_{opt}$ is not known by the users and the selections, performed by the BS, are informed to the users via notification as proposed in Sec. III-A3. The results consider blocks of $3,200$ signal vectors (19,200 bits) transmitted to each user with a notification of $F_{notf} = 10$ repetitions $(40\,\text{bits})$ at the beginning of each block. In the simulation, before the transmission of a new signal vector block, a new channel matrix sample is generated and kept fixed throughout this block. The coincidence of the performance curves obtained with error-free notification shows the effectiveness of the proposed method of notification. For the BD system case (Fig. 3b), the dashed curve, $\mathbf{Q}$-SNR-Est, shows performance results when the estimate $\hat{\mathbf{J}}^{BD}_k$ is used for signal detection. The incurred performance loss was of 0.25 dB approximately. BER performance curves with the IBP matrix selection based on the maximization of achievable rate, $\mathbf{Q}$-Rate, and the semi-analytic error probability upper bound, $\mathbf{Q}$-SNR-UB, for both precoders, are also included in Fig. 3.

The BER performance gain incurred by the use of $SNR^k_{r;bit}$ maximization strategy for IBP encoding matrix selection, while varying the number of information bearing positions is evaluated. In Figs. 4 and 5, the configuration $\{10, (5, N_{ibp}), 2\}$, for values of N_{ibp} varying from 1 to $N_r - 1$ was considered. In Fig. 5a, when using a single information bearing position ($N_{ibp} = 1$) was used, an expressive performance gain was observed. The behavior is credited to, in the case $N_{ibp} = 1$, a single column of $\mathbf{J}^{BD}_k$, e.g. $\|\mathbf{j}^k_i\|^2 = \lambda^k_i$, will be in use. In this configuration, a null entry will exist in all mean vectors $\bar{\mathbf{q}}(l)$ associated to each $\mathbf{Q}_l \in \mathcal{Q}$. And, following the maximization in (59) for IBP encoding matrix selection, this zero entry will permanently be associated to the lowest eigenvalue of $\mathbf{J}^{BD}_k$. The consequence of this is that detections with the smallest SNR values (the lowest value in $\mathbf{v}_k$) will be avoided. Fig. 5b focuses on the results for values of N_{ibp} varying from 2 to $N_R - 1$, where a significant performance gain of 2.5 dB can be observed for $N_{ibp} = 2$.

Figs. 6a and 6b present maximum sum-rate results considering ZF- and BD-precoded systems, respectively, for $\{8, (4, 2), 2\}$ e $\{10, (5, 2), 2\}$ configurations. These results are obtained for distinct IBP encoding matrix selection strategies: random selection ($\mathbf{Q}$-Rand), $SNR^k_{r;bit}$ maximization ($\mathbf{Q}$-SNR), and sum-rate maximization ($\mathbf{Q}$-Rate).

In Fig. 6a, the proposed IBP coding matrix selection strategies for ZF-precoded system achieve higher sum-rate values compared to the random selection of $\mathbf{Q}^k$. On the other hand, when BD-precoded system is considered, the proposed strategies are not able to increase the sum-rate.

As observed in Figs. 3a and 6a, the IBP encoding matrix selection criteria based on $SNR^k_{r;bit}$ or sum-rate maximization, in (41) and (45) respectively, result on the same sum-rate and detection performance for ZF-precoded system. This is due to the equivalence observed between the strategies for the configurations presented in these numerical examples. Analyzing the sum-rate maximization criterion in (45), for the presented configurations, the set of L vectors $\bar{\mathbf{q}}$ that compose the hypotheses to test in the maximiza-

TABLE I: System characteristics of $\{N_t; (N_r; N_{ibp}); K\}$.

$\{8; (4; N_{ibp}); 2\}$					$\{10; (5; N_{ibp}); 2\}$				
N_{ibp}	C_t	M_{pos}	R	L	N_{ibp}	C_t	M_{pos}	R	L
1	4	4	8	1	1	5	4	8	5
2	6	4	12	15	2	10	8	14	45
3	4	4	16	1	3	10	8	18	45
4	1	1	16	1	4	5	4	20	5
					5	1	1	20	1

(a) (b)

Fig. 2: BER performance of MU-PIM-MIMO system with $\{8; (4; N_{ibp}); 2\}$ when using randomly chosen IBP encoding matrices, Q_i-Rand. (a) ZF-precoding. (b) BD-precoding.

(a) (b)

Fig. 3: BER performance of MU-PIM-MIMO system with $\{8; (4; 2); 2\}$. (a) ZF-precoding. (b) BD-precoding.

tion derive from a unique vector, here called basis vector, but with permuted elements. Evaluating the terms I_k that compose the objective function I_T of the optimization problem, I_k and I_j, $j \neq k$, are summations comprised of the same terms, but permuted. Thus, for all single basis vectors configurations, $I_1 = I_2 = \dots = I_K$. As a consequence, the sum-rate maximization problem reduces to the minimization of I_j that is equivalent to the maximization of $SNR^k_{r,bit}$ for the ZF-precoded system.

V. IBP ENCODING MATRIX AND ANTENNA SELECTION

A. Communication using a subset of the transmit antennas

In transmit antenna selection, the base station is equipped with N_{ta} RF chains and N_t

Fig. 4: BER performance of MU-PIM-MIMO system with $\{10; (5; N_{ibp}); 2\}$ with ZF-precoding.

is the total number of antennas available for transmission, $N_{ta} < N_t$. In this configuration, the length of the precoded data vector x equals the number of RF chains and the possibility of assigning the entries of this vector to the most favorable transmit antennas is introduced. The number of possible choices, S_{Tx}, to allocate precoded data vectors to transmit antennas equals to $S_{Tx} = \binom{N_t}{N_{ta}}$.

To each possible subset of the transmit antennas, corresponds a vector $t \in \{0; 1\}^{N_t \times 1}$, where the value 1 in the nth entry indicates that the nth antenna is active, whereas the value 0 represents that this antenna is off. The possible t values are gathered in the set $T = \{t_i\}_{i=1}^{S_{Tx}}$.

The subchannel matrix, $H_{(t)} \in \mathbb{C}^{KN_r \times N_{ta}}$, obtained from the column selection of $H \in \mathbb{C}^{KN_r \times N_t}$ indicated by t, is given by

$$H_{(t)} = HG_{(t)}; \qquad (66)$$

where the $G_{(t)}$, of size $N_t \times N_{ta}$, contains the columns of I_{N_t} indicated by the nonzero values of t. Thus, $G_{(t)}$ matrix complies the properties $G^T_{(t)} G_{(t)} = I_{N_{ta}}$ and $G_{(t)} G^T_{(t)} = D(t)$.

The quantities presented in Sec. II and III are calculated in the following, considering $H_{(t)}$. Their dependency on the transmit antenna vectors are denoted by the subscript (t).

B. IBP encoding matrix and transmit antenna selection

1) ZF precoder: Transmit antenna and IBP coding matrix selection is based on $SNR^k_{r,bit}$ maximization in (37), where is dependent on the transmit antenna vector, denoted as $_{(t)}$, and given by

$$_{(t)} = \sum_{m=1}^{K} \varepsilon_m g^T_{m(t)} \bar{q}^m; \qquad (67)$$

with

$$\begin{aligned} g_{(t)} &= \left[g^T_{1(t)} \; g^T_{2(t)} \; \cdots \; g^T_{K(t)} \right]^T \\ &= d\left(P_{(t)ZF}{}^H P_{(t)ZF} \right) \\ &= d\left(\left(HD(t)H^H \right)^{-1} \right): \qquad (68) \end{aligned}$$

Then, the joint $\{Q^k\}_{k=1}^{K}$ and t optimization problem is given by

$$\left(Q^1_{sel,ZF}; Q^2_{sel,ZF}; \ldots; Q^K_{sel,ZF}; t^{ZF}_{sel} \right) = \underset{\substack{Q^1; Q^2; \ldots; Q^K \in Q \\ t \in T}}{\arg\min} \; _{(t)}: \qquad (69)$$

a) Optimal joint selection: The optimization (69) is carried out by the computation of (67) for each active transmit antenna selection $t_i \in T$; $i = 1; 2; \ldots; S_{Tx}$. For a supposed t_i, the IBP encoding matrix index associated to the kth user, is given by

$$l^{ZF}_i(k) = \underset{l \in \{1; 2; \ldots; L\}}{\arg\min} \; g^T_{k(t_i)} \bar{q}(l); \qquad (70)$$

with $g_{k(t_i)}$ given in (68), for $t = t_i$. The K IBP encoding matrix indices $l^{ZF}_i(k)$ are considered in the computation of $_{(t)}$, for the supposed t_i, as

$$_{i(t_i)} = \sum_{m=1}^{K} \varepsilon_m g^T_{m(t_i)} \bar{q}(l^{ZF}_i(m)): \qquad (71)$$

Thus, the minimization of (69) results in the optimized selection of the transmit antennas, t^{ZF}_{sel}, given by

$$t^{ZF}_{sel} = t_{i^{ZF}}; \qquad (72)$$

and the selection of the kth user IBP encoding matrix $Q^k_{sel,ZF}$ is given by

$$Q^k_{sel,ZF} = Q_{l_{i^{ZF}}(k)}; \qquad (73)$$

(a) (b)

Fig. 5: BER performance of MU-PIM-MIMO system with $\{10; (5; N_{ibp}); 2\}$ and BD-precoding. (a) $1 \le N_{ibp} \le 4$, (b) $2 \le N_{ibp} \le 4$.

(a) (b)

Fig. 6: Sum-rate of MU-PIM-MIMO system for $\{8; (4; 2); 2\}$ and $\{10; (5; 2); 2\}$. (a) ZF-precoding. (b) BD-precoding.

where the index i^{ZF} is obtained by choosing the minimal $\epsilon_{(t)}$, among the S_{Tx} possibilities calculated in (71), as

$$i^{ZF} = \arg\min_{i \in \{1; 2; \ldots; S_{Tx}\}} \epsilon_{i(t_i)}; \qquad (74)$$

and the index $l_{i^{ZF}}(k)$ was previously calculated in (70).

b) Relaxed selection: The selections of N_{ta} transmit antennas and K IBP coding matrices are made separately. The selection of t precedes the users' Q^k matrix selection. The isolated selection of t is based on the minimization of $\epsilon_{(t)}$ in (67), but the influence of the IBP encoding matrices is removed in the optimization process by assigning $\bar{q} := 1_{N_r}$. This assignment is equivalent to the assumption that a hypothetical Q matrix comprised of just a single IBP pattern was employed. In this pattern, all N_r elements conveys information, or equivalently, a conventional (non-PIM) MU-MIMO system was employed. Therefore, t^{ZF}_{sel} is given by (72), where the index $i^{ZF}(k)$ in the relaxed selection is defined as:

$$\begin{aligned} i^{ZF} &= \arg\min_{i \in \{1; 2; \ldots; S_{Tx}\}} g^T_{(t_i)} 1_{KN_r} \\ &= \arg\min_{i \in \{1; 2; \ldots; S_{Tx}\}} \left(HD(t_i)H^H\right)^{-1} : \end{aligned} \qquad (75)$$

Defining the pattern t^{ZF}_{sel}, the kth IBP encoding matrix selection, $Q^k_{sel,ZF}$, is done according to (73), where the index $l^{ZF}(k)$ is obtained by

$$l^{ZF}(k) = \arg\min_{l \in \{1; 2; \ldots; L\}} g^T_{k(t^{ZF}_{sel})} \bar{q}(l); \qquad (76)$$

with $g_{(t^{ZF}_{sel})}$ given by (68), for $t = t_{sel}$.

2) BD precoder: In terms of computational cost, the BD precoder implementation requires two SVDs per user in the system, as presented in Sec. III-B1. The addition of transmit antenna selection, resulting in the joint selection of t e Q, requires two SVDs per user for every transmit antenna vector belonging to $\mathcal{T}$, to each possible user IBP encoding matrix, turning the computational cost extremely high. This motivates the adoption of relaxed selection, analogous to the ZF-precoded system.

a) Relaxed selection: The selections of the transmit antennas and IBP encoding matrix are based on the maximization of $SNR^k_{i,bit}$ in (57). The vector $v_{k(t)}$, that makes explicit the dependency on the transmit antenna vector, is given by

$$
\begin{aligned}
v_{k(t)} &= d \quad {}^{0}_{k(t)} \quad {}^{2} \\
&= \left[\; {}^{k}_{(t)1} \quad {}^{k}_{(t)2} \; \cdots \; {}^{k}_{(t)N_r} \right]^T : \qquad (77)
\end{aligned}
$$

As the transmit antenna selection impacts on the $SNR^k_{i,bit}$ values of all users, the strategy proposed here is based on the maximization of $\sum_{k=1}^{K} SNR^k_{i,bit}$, or equivalently, the maximization of

$$
\varphi_{(t)} = \sum_{m=1}^{K} v^T_{m(t)} \bar{q}^m : \qquad (78)
$$

As in the ZF-precoded case, relaxed selection of BD-precoded systems is done by the selection of t followed by the selection of the IBP encoding matrices. The selection of t is based on the maximization of $\varphi_{(t)}$ in (78), but similarly to ZF-precoded system, in the selection of the K matrices Q, the assignment $\bar{q} := 1_{N_r}$ is used. Then, t^{BD}_{sel} is given by

$$
t^{BD}_{sel} = t_{i^{BD}}; \qquad (79)
$$

where the index i^{BD} is calculated as

$$
i^{BD} = \arg\max_{i \in \{1;2;\ldots;S_{Tx}\}} \sum_{m=1}^{K} v^T_{m(t_i)} 1_{N_r}; \qquad (80)
$$

with $v_{m(t_i)}$ given by (77) for $t = t_i$.

The selected transmit antenna combination t^{BD}_{sel} is considered for the determination of the IBP encoding matrices. The kth user matrix, $Q^k_{sel,BD}$, is given by

$$
Q^k_{sel,BD} = Q_{l_{BD}(k)}; \qquad (81)
$$

where the index $l^{BD}(k)$ is given by

$$
l^{BD}(k) = \arg\max_{l \in \{1;2;\ldots;L\}} v^T_{k(t^{BD}_{sel})} \bar{q}(l); \qquad (82)
$$

with $v_{k(t^{BD}_{sel})}$ given by (77) for $t = t_{sel}$.

C. Selection of t with search-space reduction

The optimal joint and the relaxed selection procedures for transmit antenna and IBP encoding matrix selection, presented in Sec. V-B, consider the exhaustive search over all S_{Tx} patterns of t. However, when a scenario with a high value of N_t is considered, the number of possible combinations S_{Tx} scales up and the search complexity increases, turning these strategies infeasible due to the amount of computational resourcers required. Based on these considerations, the reduced-space search Iterative Search (ITES) algorithm is presented.

The non-exaustive search algorithm ITES is embedded in the transmit antenna selection of the optimization algorithms presented in Sec. V-B. ITES algorithm was originally proposed for the alocation of pilot symbols of OFDM systems, in [13], and showed to be effective to perform antenna selection in MU-MIMO communication [14]. The algorithm structure employed to the optimal joint selection of ZF-precoded system is shown in Algorithn 1, and the adaptation to the relaxed selections in ZF and BD-precoded systems is straightforward.

The algorithm is initialized with t_0, randomly selected from $\mathcal{T}$, a temporary assignment to the optimized vector t_{sel}. For t_0, $_{0(t_0)}$ is calculated, and then the temporary selection of the IBP encoding matrix IBP Q^k is determined, indicated by the index $l_0(n)$.

In the following, vectors e are generated. The entries of and are the active and non-active antenna indices of t_{sel}, respectively. The algorithm generates new possible vectors by changing, at every iteration, the position of an active antenna independently. Doing so, $N_d = N_t \; N_{ta}$ new possible t are obtained and gathered in the set $_j$. This means, each element $t_i = t^{\;j! \;\; i}$ of $_j$ is generated deactivating the jth antenna and activating the ith antenna. Among the new possible t 2 $_j$, the one that results in the minimum value of $_{(t)}$ is selected, and then the IBP encoding matrices Q^k are determined.

One cycle corresponds to N_d tests of t_i 2 $_j$ and one iteration corresponds to the search through all N_{ta} sets $_j$. Then, each iteration comprises $N_{ta} \; N_d$ tests of t. The procedure stops if a smaller $_{(t)}$ value is not found.

Algorithm 1: Selection of t using ITES.

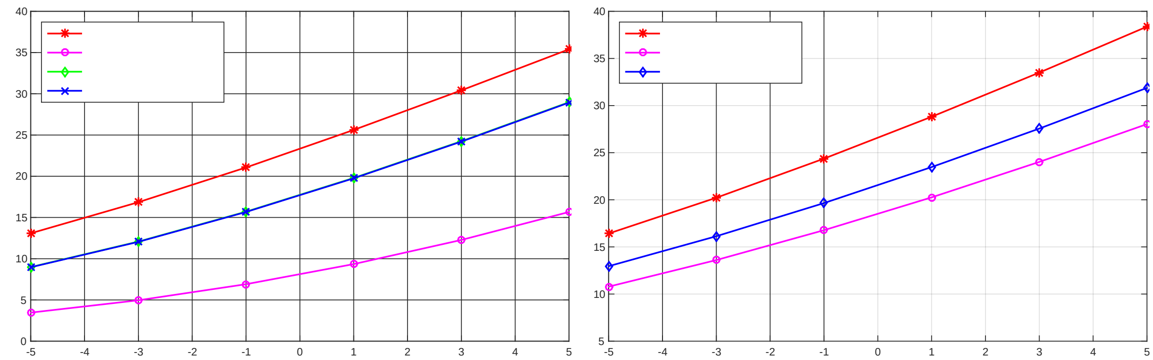
```
Data: t_0; 0(t_0)
Result: t_sel^ZF
Initialization: t_sel = t_0, in(t) = 0(t_0),
    out(t) = 1 +
  ! active antenna indices N_ta in t_sel
  ! non-active antenna indices
    N_d = N_t  N_ta in t_sel
while in(t) < out(t) do
    out(t) = in(t)
    for j = 1 to N_ta do
        t_al = t_sel
        j = { t_i^(j! i) }_{i=1}^{N_d}
        i^ZF = arg min_{i 2f 1;2;:::;N_d g} i(t_i)
        if i(t_i) < in(t) then
            in(t) = i(t_i)
            t_sel^ZF = t_{i^ZF}
        end
        Update  e  to the following
          cycle.
    end
    Store t_sel^ZF to the following iteration.
end
```

VI. NUMERICAL RESULTS - IBP ENCODING MATRIX AND TRANSMIT ANTENNA SELECTION

In this section, BER results experienced by the users are presented for ZF- and BD-precoded systems when the transmitting antennas and the encoding matrix are selected. The entries of the matrices H_k; $k = 1; 2; :::; K$ are modeled as described in Sec. IV. Results are obtained after 1; 000 channel realizations. In the case of BER, 19,200 bits transmitted to each user in each realization. Performance results are expressed in terms of (17). System configurations that employ antenna selection are specified by the set $f(N_t; N_{ta}); (N_r; N_{ibp}); Kg$.

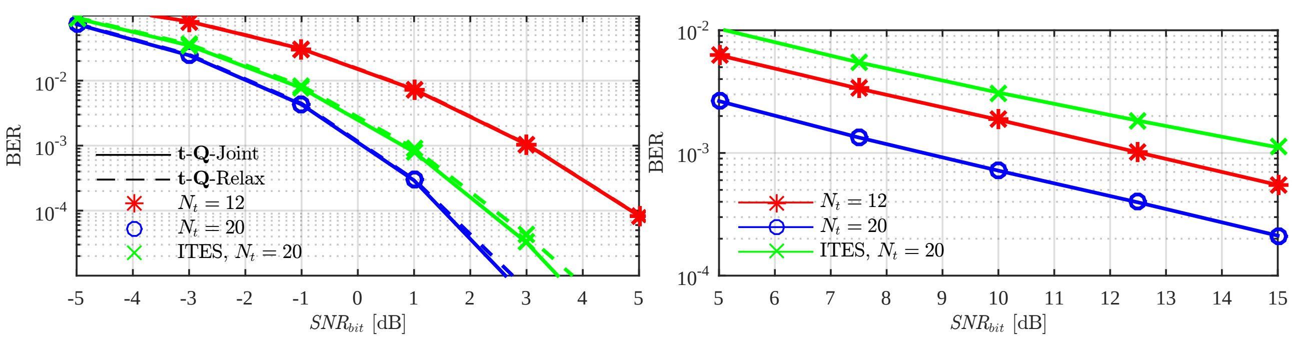
Fig. 7 shows the system sum-rate as a function of SNR_{bit}, considering the antenna subset and IBP encoding matrix selections for ZF and BD precoded systems. System configuration $f(12; 8); (4; 2); 2g$ was considered to produce the curves, identified by t-Q-Joint and t-Q-Relax in the figures. These curves are compared to scenarios where the number of antenna elements equals the number of radiofrequency chains, thus no transmit antenna selection is possible. In this case, $fN_t; (N_r; N_{ibp}); Kg = fN_t; (4; 2); 2g$ configuration is employed. In both curves, sum-rate increase is observed compared to the configuration without antenna selection and $N_t = 8$. Therefore, it reflects the sum-rate gain achieved by the possibility of antenna selection. The curve without antenna selection and $N_t = 12$ serves as a benchmark to the antenna selection strategies, if all antenna elements were connected to radiofrequency chains. The proximity of the antenna and IBP encoding matrix selection strategy curves, specially in the ZF-precoded system, shows the effectiveness of our proposal. At last, in ZF-precoded systems, the relaxed optimization results shows no noticeable loss compared to the optimal joint optimization curve.

Fig. 8 exhibits, for $f(N_t; 8); (4; 2); 2g$, the performance of ZF and BD precoded systems for different values of the number of transmit antennas available, N_t, and the other parameters kept fixed. Fig. 8a shows the bit-error rate in the joint selection procedure, t-Q-Joint, and the relaxed procedure, t-Q-Relax, for ZF-precoded system. Fig. 8b illustrates the bit-error rate of the relaxed selection for BD-precoded system. In these figures, nearly coincident curves of the optimal and relaxed selection are observed for



(a) (b)

Fig. 7: Sum-rate as a function SNR_{bit} in MU-PIM-MIMO systems. System configurations: with antenna selection (**t**-**Q**-Joint, **t**-**Q**-Relax): $f(12; 8); (4; 2); 2g$, without antenna selection: $fN_t; (4; 2); 2g$. (a) ZF-precoding with joint, t-Q-Joint, and relaxed selection, t-Q-Relax. (b) BD-precoding with relaxed selection, t-Q-Relax.



(a) (b)

Fig. 8: BER performance of MU-PIM-MIMO system with $f(N_t; 8); (4; 2); 2g$. (a) ZF-precoding with joint, t-Q-Joint, and relaxed selection, t-Q-Relax. (b) BD-precoding with relaxed selection.

ZF-precoded system. This evidences the attractiveness of the relaxed implementation due to the lower computational cost, as shown in Table II [15], [16], with the computational cost expressed by the number of floating point operations required by the different selection procedures to generate the selected t and Q. Observing the BD case, a performance gain of approximately 2 dB is achieved when the number of antennas available at the transmitter is increased from 12 to 20.

Comparing the isolated selection of the IBP encoding matrix, exhibited in Fig. 3, to the selection of both transmit antenna combination and IBP encoding matrix, shown in Fig. 8, results in a very significant performance gain obtained by the latter, although the number of active transmit antennas in both configurations remain unaltered ($N_t = 8$ in Fig. 3 and $N_{ta} = 8$ in Fig. 8).

Additionally, in Figs. 8a and 8b, the use of ITES algorithm to the selection of the active transmit antennas results in performance loss of approximately 1 dB and 6 dB compared to the full search-space strategies for ZF and BD-precoded systems, respectively. However the ITES-based strategies achieved substantial computational complexity savings as shown in Table III. In terms of search-space reduction, ITES

TABLE II: Computational cost / t and Q selection for $f(20; 8); (4; 2); 2g$.

Selection Method	ZF	BD
Exhaustive search, Joint	423; 700; 093:5	-
Exhaustive search, Relaxed	390; 570; 235:5	777; 172; 165:5
ITES, Joint	2; 459; 92:54	-
ITES, Relaxed	2; 397; 850:3	4; 692; 769:2

TABLE III: relative computational cost reduction / t and Q selection for $f(20; 8); (4; 2); 2g$.

	ZF	BD
Exhaustive search: Relaxed vs Joint	7:82%	-
ITES: Relaxed vs Joint	2:52%	-
ITES, Joint vs Exhaustive search, Joint	99:41%	-
ITES, Joint vs Exhaustive search, Relaxed	99:37%	-
ITES, Relaxed vs Exhaustive search, Joint	99:43%	-
ITES, Relaxed vs Exhaustive search, Relaxed	99:39%	99:40%

resulted in the reduction of the full search-space T, of cardinality $125; 970$, to a reduced space of 731 and 767 hypotheses in the joint and relaxed selections for ZF-precoded system, respectively, and 368 and 379 when joint and relaxed selections for BD-precoded systems, all considering $N_t = 20$.

VII. Conclusion

This article considers a multiuser MIMO system that uses PIM to perform communication, named here MU-PIM-MIMO. In MU-PIM-MIMO, part of the information to be transmitted is encoded in the choice of the IBP precoder matrix, which, in turn, is used set the entries of the information vector as information-bearing positions or null-valued positions. The modeling of the system was presented, and expressions of the energy relations were developed. Based on this mathematical model, expressions for the user achievable rate and a semi-analytical upper bound of the error probability were derived. Then, particularizations of the expressions for the two considered precoders, namely, ZF and BD, were presented. For each of these precoders, the influence of the choice of the IBP encoding matrix was studied and strategies for selecting the most adequate IBP matrix were derived. As seen, strategies based on SNR maximization and the achievable rate maximization were presented for both ZF and BD precoders. In addition to the sole selection of IBP encoding matrices, the availability of extra antenna elements at the transmitter was considered. The selection of the subset of transmit antennas to communicate, together with the IBP encoding matrix, has shown to result in very significant detection performance gains. As the joint antenna and IBP matrix selection may result in prohibitively complex strategies, relaxed versions of those strategies result in attractive balance between detection performance and computational complexity. An extensive set of numerical results, considering these different strategies being applied on MU-PIM-MIMO systems with different configurations was presented, and evidences the effectiveness of the proposed techniques.

References

[1] E. Basar, M. Wen, R. Mesleh, M. Di Renzo, Y. Xiao, and H. Haas, "Index modulation techniques for next-generation wireless networks," *IEEE Access*, vol. 5, pp. 16 693–16 746, 2017. doi: 10.1109/ACCESS.2017.2737528

[2] H. Zhang, M. Ma, and Z. Shao, "Multi-user linear precoding for downlink generalized spatial modulation systems," *IEEE Communications Letters*, vol. 24, no. 1, pp. 212–216, 2019. doi: 10.1109/LCOMM.2019.2947451

[3] N. Souto and A. Correia, "A precoding aided space domain index modulation scheme for downlink multiuser mimo systems," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 10, pp. 12 333–12 337, 2020. doi: 10.1109/TVT.2020.3012044