Error Probability of Multichannel Reception with θ-QAM Scheme Under Correlated Nakagami-m Fading

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***Abstract*—This paper presents a unified analysis for the compact receiver with spatial diversity. The maximum ratio combining (MRC) receiver for linear antenna arrays is considered with the modulation scheme θ-QAM. The mathematical development takes into account parameters related to the physical structure of the array, as well, as the probability distribution used for modeling the direction of arrival (DoA) of the signals in the array. Von Mises and Gaussian distributions are considered to characterize the DoA. The unification of all the results in exact expressions to evaluate the symbol error probability (SEP) at the output of a MRC receiver is the main contribution of the research.**

***Index Terms*—Multichannel reception, θ-QAM modulation, Correlated Nakagami-m fading.**

I. Introduction

In most diversity techniques detection errors may occur at the receiver when the transmission medium imposes a severe attenuation on the transmitted signals. Thus, if it is possible to provide the receiver with different copies of the transmitted signal, from different independent sub-channels, the probability that the transmitted copies fade simultaneously is reduced. For instance, if p is the probability that a copy of the transmitted signal fades below a critical level, then p^L is the probability that all the copies in L independent sub-channels decrease below the same critical level.

There are many techniques that provide the receiver with different copies of the transmitted signal, which include frequency diversity [1], in which the information signal modulates a number of carriers with different frequencies separated by a bandwidth that is greater than or equal to the coherence bandwidth of the wireless channel.

Another diversity scheme is spatial diversity, in which multiple antennas are used for transmission and reception of the signal. The antennas in the receiver structure are generally separated to assure that signals coming from different sub-channels fade independently. Usually, in shadowing environments, the distance from one antenna to nearest one must be at least ten wavelengths, to guarantee independent fading between the channels. In urban areas, in which the variations in the envelope of the transmitted signal can be modeled by a Rayleigh distribution, a separation between the antennas of only half wavelength suffices [2].

Depending on the transmission frequency and the size of the receiver structure, it may not be possible to guarantee independent fading between the samples captured by the elements of the antenna array. For those compact receivers the diversity gain is affected by the correlation between the signals captured by the different elements of the array.

The evaluation of compact receivers, under weakly correlated fading, can be adequately performed by the use of the Nakagami-m distribution [3]–[6], which has been employed to model variations in the intensity of signals transmitted in urban environments. Besides the Doppler effect [7], it has been observed in those environments that the signals that arrive at the elements of the array are composed by the sum of the signals that come from different directions and the groups of those signals are usually known as sub-paths. The correlation between the received signals in compact receivers can be taken into account by using well known mathematical results, as presented in [8], [9].

In this paper a unified analysis is proposed for the compact receiver with spatial diversity. The maximum ratio combining (MRC) receiver for linear antenna arrays is considered, with the modulation scheme θ-QAM proposed in [10], [11]. It is worth mentioning that θ-QAM includes schemes such as Square Quadrature Amplitude Modulation (SQAM) and Triangular Quadrature Amplitude Modulation (TQAM), which are used in applications that range from synchronous transmission modems, with various constellations (16-QAM, 64-QAM and 256-QAM), to digital television, Digital Video Broadcasting Terrestrial (DVB-T) and International Standard for Digital Television (ISDB-T), in which the data are coded and arranged in blocks of 188 bytes, and sent to a segmenter and then arranged in one, two or three coding blocks, which are called layers. The data are then mapped in symbols of 16-QAM or 64-QAM constellations.

The performance of antenna arrays with an MRC has been addressed in the literature (e.g. [12]–[16]), but, to the best of the authors' knowledge, the performance considering physical parameters of the array in the scenario of θ-QAM modulation has not been covered. The mathematical development presented in this paper takes into account parameters related to the physical structure of the array as well as the probability distribution used for modeling the direction of arrival (DoA) of the signals in the array. Von Mises and Gaussian distributions

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are considered to characterize the DoA. The unification of the results in exact expressions to evaluate the symbol error probability (SEP) at the output of a MRC receiver is the main contribution of the paper. It is worth mentioning that an important aspect of the Von Mises distribution is the fact that it models anisotropic propagation processes in a better way, as compared to the uniform distribution, and leads to mathematical expressions for the spatial correlation coefficients that are simpler than the ones obtained with Gaussian and cosine distributions [17].

II. Characterization of the Problem

In communication theory, the early applications of the multivariate analysis were presented in [18]–[20], to analyze the capacity of multiple antenna narrow band systems [21], [22], in the study of the optimum signal-to-interference plus noise ratio (SINR) that can be obtained by linear detectors for multiple users in CDMA systems, in the study of the near-far interference in CDMA systems and in the analysis of the spectral efficiency of CDMA.

One of those applications is the spatial processing of signals by means of linear antenna arrays. Those arrays can be used in base stations as well as in compact receivers for mobile terminals. The use of compact receivers is usual in modern communication systems. With the dissemination of wideband multiple access systems, an ever increasing range of services is provided to the users [23]. Such services can suffer a decrease in quality as a result of characteristics of urban and suburban propagation environments, which affect parameters such as error probability and spectral efficiency. One of those characteristics inherent to urban environments, mainly big metropolis environments, is the fading, usually modeled by the Nakagami distribution.

In [8], it was shown that, in a correlated Nakagami fading channel, the signal received in the k-th branch of a L-branch receiver can be written as

$$r_k(t) = s(t)A_k e^{j\phi_k} + n_k(t), \quad k = 1, 2, \ldots, L, \tag{1}$$

in which $s(t)$ represents the transmitted signal and $n_k(t)$ represents the k-th sample of an independent and identically distributed complex Gaussian noise with zero mean and variance $\frac{N_0}{2}$ per dimension. The phase ϕ_k in uniformly distributed in $[0, 2\pi)$ and A_k is the fading envelope characterized by a Nakagami distribution with probability density function (pdf) given by

$$p_{A_k}(A_k) = \frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)^{m_k} {A_k}^{2m_k-1} e^{-\left(\frac{m_k}{\Omega_k}\right)} {A_k}^2, \tag{2}$$

in which $A_k > 0$, $m_k = \Omega_k^2/\text{Var}[A_k^2]$, known as the fading factor, models the intensity of the fading of the signal that is captured by the k-th branch of the receiver, $\Omega_k = E[A_k^2]$ models the mean power of the signal in the k-th branch, $E[\cdot]$ is the expectation operator and Var$[\cdot]$ is the variance of the argument.

Assuming that the multiple branch receiver has maximum ratio combining, it follows that, if the fading is flat and the phase variation imposed by the channel is known, then the signal to noise ratio (SNR) in the receiver output, denoted by γ, can be written as

$$\gamma = \frac{E_s}{N_0}\sum_{k=1}^{L} A_k^2 = \sum_{k=1}^{L}\gamma_k, \tag{3}$$

in which $\gamma_k = \left(\frac{E_s}{N_0}\right) A_k^2$ and E_s is the symbol energy.

In [8] it is shown that the characteristic function $\Phi_\gamma(\omega)$ of the random variable γ that represents the SNR at the L-branch MRC receiver, for a fading modeled by a Nakagami-m pdf, can be written as [8]

$$\phi_\gamma(\omega) = \left|\boldsymbol{I}_L - j\omega\frac{\overline{\gamma}}{m}\boldsymbol{\Delta}\right|^{-m}, \tag{4}$$

in which $j = \sqrt{-1}$, $|\cdot|$ represents the matrix determinant operator, $\boldsymbol{I}_L$ is the identity matrix of order L and $\boldsymbol{\Delta}$ is the covariance matrix of complex elements, which is obtained from the covariance matrix [8], [9]

$$\boldsymbol{\Sigma} = \sigma^2 \begin{bmatrix} a_1 & 0 & b_{12} & \beta_{12} & \cdots & b_{1L} & \beta_{1L} \\ 0 & a_1 & \beta_{21} & b_{21} & \cdots & \beta_{L1} & b_{L1} \\ b_{21} & \beta_{21} & a_2 & 0 & \cdots & b_{2L} & \beta_{2L} \\ \beta_{12} & b_{12} & 0 & a_2 & \cdots & \beta_{L2} & b_{L2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ b_{L1} & \beta_{L1} & b_{L2} & \beta_{L2} & \cdots & a_L & 0 \\ \beta_{1L} & b_{1L} & \beta_{L2} & b_{2L} & \cdots & 0 & a_L \end{bmatrix}, \tag{5}$$

in which $\sigma^2 = E[x_k x_k] = E[y_k y_k]$, $a_k = E[x_k^2]/\sigma^2 = 1$, $b_{kp} = E[x_k x_p]/\sigma_2 = b_{pk}$, $\beta_{kp} = E[x_k y_k]/\sigma^2 = -\beta_{pk}$. The Cartesian representation of the complex number r_k is (x_k, y_k). Thus, the $2L$-dimensional vector $\boldsymbol{r}$ which represents the samples of received signal at the L-branch antenna array can be expressed as

$$\boldsymbol{r}^T = [x_1\ y_1\ x_2\ y_2\ \cdots\ x_L\ y_L], \tag{6}$$

in which T indicates the transpose operation.

In a mobile communications environment, the signal samples, (x_i, y_i), captured at the i-th element of the linear array, are Gaussian random variables resulting from the sum of a considerable number of samples of signals that come from multipath propagation between the transmitting antenna and the i-th branch of the MRC receiver.

The matrix $\boldsymbol{\Delta}$, which is obtained from the matrix $\boldsymbol{\Sigma}$, is given by [8, Eq. 14]

$$\boldsymbol{\Delta} = \sigma^2 \begin{bmatrix} a_1 & B_{12}^* & B_{13}^* & \cdots & B_{1L}^* \\ B_{12} & a_2 & B_{23}^* & \cdots & B_{2L}^* \\ B_{13} & B_{23} & a_3 & \cdots & B_{3L}^* \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_{1L} & B_{2L} & B_{3L} & \cdots & a_L \end{bmatrix} = \sigma^2\boldsymbol{\Lambda}, \tag{7}$$

in which $B_{kl} = b_{kl} + j\beta_{kl}$ and * indicates the conjugation.

The average bit error probability of the reception system under fading can be determined by averaging the error probability conditioned to the signal to noise ratio γ at the output of the receiver, $P(e|\gamma)$, by the pdf of the signal to noise ratio, $p_\gamma(\gamma)$ [1],

$$P_e = \int_0^\infty P(e|\gamma)p_\gamma(\gamma)d\gamma. \tag{8}$$

Hence, if $p_\gamma(\gamma)$ is the pdf of the signal to noise at the output of the MRC receiver and $P(e|\gamma)$ is the symbol error probability conditioned to that SNR, then the expression P_e in Equation 8 gives the average symbol error probability of the MRC receiver. The derivation of that probability for a θ-QAM scheme is presented in Section II-A.

A. Symbol Error Probability of a θ-QAM Scheme under Correlated Nakagami-m fading

Since the SNR at the output of a MRC receiver is γ, and given that the error probability conditioned to that SNR, $P(e|\gamma)$ is known, the average probability obtained by using the pdf of γ, under Nakagami-m fading, denoted by $p_\gamma(\gamma)$, is presented in Equation 8. To analyze the receiver with a θ-QAM scheme, by means of the symbol error probability (SEP), one must use the expression $P(e|\gamma)$ of θ-QAM under Gaussian noise, given by Equation 9.

$$\begin{aligned}P(e|\gamma) =& c_1c_2\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+\theta}{2}} e^{-\gamma\delta^2\csc^2(\phi)}d\phi \\ &+ c_1c_3^2\int_{\theta}^{\pi-\theta} e^{-\gamma\delta^2\csc^2(\phi)\sec^2\left(\frac{\theta}{2}\right)\sin^2(\theta)}d\phi \\ &+ c_1c_4\int_{\frac{\pi-\theta}{2}}^{\pi} e^{-\gamma\delta^2\csc^2(\phi)}d\phi \\ &+ c_1c_5\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+2\theta}{2}} e^{-\gamma\delta^2\csc^2(\phi)}d\phi \\ &+ c_1c_6\int_{2\theta}^{\pi} e^{-4\gamma\delta^2\csc^2(\phi)\sin^2(\theta)}d\phi.\end{aligned} \tag{9}$$

The parameters c_1, c_2, c_3, c_4, c_5 and c_6, regarding the geometry of the constellation θ-QAM, are given by [10]

$$\begin{array}{ll} c_1 = \frac{1}{2\pi M} & c_2 = 4(\sqrt{M}-2)(\sqrt{M}-1) \\ c_3 = \sqrt{2}(\sqrt{M}-1) & c_4 = 5(\sqrt{M}-2)+6 \\ c_5 = 3(\sqrt{M}-2)+2 & c_6 = \sqrt{M}, \end{array} \tag{10}$$

in which M is the number of symbols $s_{m,n}$ of the constellation θ-QAM, whose coordinates, (s_xi, s_yj), are given by [24]

$$\begin{aligned} s_xi &= \left[2(j-1)+1-\sqrt{M}\right]D + [2\text{mod}(i,2)-1]\frac{a}{2} \\ s_yj &= -\left[2(i-1)+1-\sqrt{M}\right]\frac{b}{2}, \end{aligned} \tag{11}$$

with $i = 1, \cdots, \sqrt{M}$, $j = 1, \cdots, \sqrt{M}$, $a = 2D\cos\theta$, $b = 2D\sin\theta$ and $\text{mod}(x,y)$ denotes the rest of the division of x by y. Half the Euclidean distance between adjacent symbols of the constellation is given by

$$D = \frac{\sqrt{6E_{\text{av}}}}{\sqrt{3M+(4-M)\cos(2\theta)}} \tag{12}$$

and is related to the parameter δ and to the mean energy by symbol, E_{av}, by the expression

$$\delta = \frac{D}{\sqrt{E_{\text{av}}}} = \frac{\sqrt{6}}{\sqrt{3M+(4-M)\cos(2\theta)}}. \tag{13}$$

The SEP of the θ-QAM scheme, weighted by the pdf of the SNR $p_\gamma(\gamma)$ at the output of the MRC receiver, for a given type of fading, can be calculated from Equation 8. Since the characteristic function of a random variable defined for positive values is given by

$$\phi_X(\omega) = \int_0^\infty e^{-j\omega x}f_X(x)dx, \tag{14}$$

Equation 8 can be rewritten in terms of the characteristic function of the variable γ, $\phi_\gamma(\omega)$, as

$$\begin{aligned}P_e =& c_1c_2\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+\theta}{2}} \phi_\gamma\left(j\delta^2\csc^2(\phi)\right)d\phi \\ &+ c_1c_3^2\int_{\theta}^{\pi-\theta} \phi_\gamma\left(j\delta^2\sec^2\left(\frac{\theta}{2}\right)\sin^2(\theta)\csc^2(\phi)\right)d\phi \\ &+ c_1c_4\int_{\frac{\pi-\theta}{2}}^{\pi} \phi_\gamma\left(j\delta^2\csc^2(\phi)\right)d\phi \\ &+ c_1c_5\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+2\theta}{2}} \phi_\gamma\left(j\delta^2\csc^2(\phi)\right)d\phi \\ &+ c_1c_6\int_{2\theta}^{\pi} \phi_\gamma\left(j\delta^2 4\csc^2(\phi)\sin^2(\theta)\right)d\phi.\end{aligned} \tag{15}$$

Using the characteristic function of the SNR γ, given in Equation 4, it is possible to rewrite Equation 15 as

$$\begin{aligned}P_e =& c_1c_2\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+\theta}{2}} \left|\boldsymbol{I}_L + \frac{\delta^2\overline{\gamma}}{m}\boldsymbol{\Lambda}\csc^2(\phi)\right|^{-m}d\phi \\ &+ c_1c_3^2\int_{\theta}^{\pi-\theta} \left|\boldsymbol{I}_L + \frac{\delta^2\overline{\gamma}}{m}\boldsymbol{\Lambda}\sec^2\left(\frac{\theta}{2}\right)\sin^2(\theta)\csc^2(\phi)\right|^{-m}d\phi \\ &+ c_1c_4\int_{\frac{\pi-\theta}{2}}^{\pi} \left|\boldsymbol{I}_L + \frac{\delta^2\overline{\gamma}}{m}\boldsymbol{\Lambda}\csc^2(\phi)\right|^{-m}d\phi \\ &+ c_1c_5\int_{\frac{\pi-\theta}{2}}^{\frac{\pi+2\theta}{2}} \left|\boldsymbol{I}_L + \frac{\delta^2\overline{\gamma}}{m}\boldsymbol{\Lambda}\csc^2(\phi)\right|^{-m}d\phi \\ &+ c_1c_6\int_{2\theta}^{\pi} \left|\boldsymbol{I}_L + 4\frac{\delta^2\overline{\gamma}}{m}\boldsymbol{\Lambda}\csc^2(\phi)\sin^2(\theta)\right|^{-m}d\phi.\end{aligned} \tag{16}$$

Ton evaluate the SEP, it is necessary to determine the spatial correlation coefficients of the matrix $\boldsymbol{\Lambda}$ presented in Equation 7. Using those coefficients, the direction of arrival of the signals at the array is taken into account. In this article the equally spaced linear array is considered, and the probability distributions used to characterize the DoA are Von Mises and Gaussian.

III. Determination of the Coefficients of the Spatial Correlation Matrix

The Von Mises distribution used to model the DoA in the determination of the spatial correlation coefficients includes other distributions, such as Gaussian and cosine distributions, and provides expressions simpler than the Gaussian and cosine distributions. Von Mises distribution was introduced in 1918 [25] and has been used as an appropriate alternative to model the DoA in urban environments [26], in which the spreading of waves is nonuniform [27]–[31]. The pdf of the Von Mises distribution is

$$p_\varphi(\varphi) = \begin{cases} \frac{e^{\kappa\cos(\varphi-\varphi_o)}}{2\pi \mathrm{I}_0(\kappa)}, & -\pi+\varphi_o \leq \varphi \leq \pi+\varphi_o \\ 0, & \text{otherwise} \end{cases}, \quad (17)$$

in which φ_o is the average direction of a set of direction components that ranges in the interval $[0,\pi)$ and $\kappa > 0$ is a parameter that influences that pdf sharpness.

Taking the two first terms of the Taylor series of $\cos(\varphi-\varphi_o)$ and using the modified Bessel function approximation [32],

$$\mathrm{I}_0(x) \approx \frac{e^x}{\sqrt{2\pi x}}, \quad x \gg \frac{1}{4}, \quad (18)$$

it is possible to obtain the approximation

$$\frac{e^{\kappa\cos(x)}}{2\pi \mathrm{I}_0(\kappa)} \approx \frac{e^{-\left(\frac{x}{\sqrt{2}\frac{1}{\sqrt{\kappa}}}\right)^2}}{\sqrt{2\pi}\frac{1}{\sqrt{\kappa}}}, \quad (19)$$

that is, a Gaussian pdf with variance $\frac{1}{\kappa}$ can be obtained from the Von Mises pdf for $\kappa \gg \frac{1}{4}$.

For the determination of the spatial correlation coefficients, it is assumed that the k-th signal sample captured in the k-th element of a linear array with L elements, equally spaced by a distance d, can be written as $e^{-jk\beta d\sin(\varphi)}$, where β is the wave number of the electromagnetic wave and is $2\pi/\lambda$.

The spatial correlation coefficients for the samples captured at two elements of a linear array of L elements equally spaced by distance d are denoted by $\rho(m,n)$ and given by [8], [9]

$$\begin{aligned} \rho(m,n) =& B_{m,n} = E[r_m r_n^*] \\ =& \frac{1}{\sqrt{P_m P_n}} \int_{-\pi+\varphi_o}^{\pi+\varphi_o} r_m(\varphi) r_n^*(\varphi) p_\varphi(\varphi) d\varphi \\ =& \rho_R(m,n) + j\rho_I(m,n), \end{aligned} \quad (20)$$

in which

$$P_m = \int_{-\pi+\varphi_o}^{\pi+\varphi_o} |r_m(\varphi)|^2 p_\varphi(\varphi) d\varphi. \quad (21)$$

Considering that the sampled signals arrive as plane waves at the antenna array elements, represented by vector $\boldsymbol{r}$ in Equation 6, one can rewrite B_{mn} as

$$B_{m,n} = \int_{-\pi+\varphi_0}^{\pi+\varphi_0} e^{j(m-n)\beta d\sin(\varphi)} p_\varphi(\varphi) d\varphi. \quad (22)$$

Using the Von Mises pdf in Equation 22, the Euler identity and the expansions in modified Bessel series of the terms $\cos((m-n)\beta d\sin\varphi)$ and $\sin((m-n)\beta d\sin\varphi)$ [32], it is possible to write the real and imaginary parts of $\rho(m,n)$ as

$$\begin{aligned} \rho_R(m,n) =& 2\sum_{l=1}^{\infty}\left(\frac{\mathrm{I}_{2l}(\kappa)}{\mathrm{I}_0(\kappa)}\right)\mathrm{J}_{2l}((m-n)\beta d)\cos(2l\varphi_o) \\ &+ \mathrm{J}_0((m-n)\beta d), \\ \rho_I(m,n) =& 2\sum_{l=0}^{\infty}\left(\frac{\mathrm{I}_{2l+1}(\kappa)}{\mathrm{I}_0(\kappa)}\right)\mathrm{J}_{2l+1}((m-n)\beta d)\cdot \\ &\sin((2l+1)\varphi_o). \end{aligned} \quad (23)$$

Those coefficients correspond, respectively, to the real and imaginary parts of the elements of the matrix presented in Equation 7, that is, $\rho_R(k,l) = b_{k,l}$ and $\rho_I(k,l) = \beta_{k,l}$.

Similarly, considering the Gaussian pdf to model the DoA,

$$p_\varphi(\varphi) = \frac{k_g}{\sqrt{2\pi\sigma_\varphi^2}} e^{-\frac{(\varphi-\varphi_o)^2}{2\sigma_\varphi^2}}, \quad -\pi+\varphi_o \leq \varphi \leq \pi+\varphi_o, \quad (24)$$

and using Equation 20, the spatial correlation coefficients can be written as

$$\begin{aligned} \rho_R(m,n) =& \frac{2k_g}{\sqrt{\pi}}\sum_{l=1}^{\infty}\mathrm{J}_{2l}\left((m-n)\beta d\right)\mathcal{R}_g(2l,\phi_o,\sigma_\phi) \\ &+ \mathrm{J}_0\left((m-n)\beta d\right), \\ \rho_I(m,n) =& \frac{2k_g}{\sqrt{\pi}}\sum_{l=0}^{\infty}\mathrm{J}_{2l+1}\left((m-n)\beta d\right)\mathcal{I}_g(2l+1,\phi_o,\sigma_\phi), \end{aligned} \quad (25)$$

in which

$$\begin{aligned} \mathcal{R}_g(a,b,c) =& \frac{\sqrt{\pi}}{2}\left[\cos(ab)\mathcal{A}(a,c) - \sin(ab)\mathcal{B}(a,c)\right]e^{-\frac{(ac)^2}{2}}, \\ \mathcal{I}_g(a,b,c) =& \frac{\sqrt{\pi}}{2}\left[\sin(ab)\mathcal{A}(a,c) - \cos(ab)\mathcal{B}(a,c)\right]e^{-\frac{(ac)^2}{2}}, \end{aligned} \quad (26)$$

$$\begin{aligned} \mathcal{A}(a,b) =& \mathrm{Re}\left\{\mathrm{erf}\left(\frac{\pi}{\sqrt{2}b} - j\frac{ab}{\sqrt{2}}\right)\right\} \\ &- \mathrm{Re}\left\{\mathrm{erf}\left(-\frac{\pi}{\sqrt{2}b} - j\frac{ab}{\sqrt{2}}\right)\right\}, \\ \mathcal{B}(a,b) =& \mathrm{Im}\left\{\mathrm{erf}\left(\frac{\pi}{\sqrt{2}b} - j\frac{ab}{\sqrt{2}}\right)\right\} \\ &- \mathrm{Im}\left\{\mathrm{erf}\left(-\frac{\pi}{\sqrt{2}b} - j\frac{ab}{\sqrt{2}}\right)\right\} \end{aligned} \quad (27)$$

and k_g is a normalization constant for the area under the Gaussian pdf,

$$k_g = \frac{1}{\mathrm{erf}\left(\frac{\pi}{\sqrt{2}\sigma_\varphi}\right)}. \quad (28)$$

The term $\mathrm{erf}(\cdot)$ represents the error function and can be written in terms of Dawson integral when its argument is complex [32]

$$\mathrm{erf}(x) = 1 - e^{-x^2}\mathrm{w}(jx) \quad (29)$$

in which

$$\mathrm{w}(x) = e^{-x^2}\left(1 + \frac{2j}{\sqrt{\pi}}\int_0^x e^{t^2}dt\right). \quad (30)$$

IV. Evaluation of the Impact of the Electromagnetic Coupling

One of the main problems caused by the use of compact antenna arrays is the appearance of the magnetic coupling between the elements. When the effect of the coupling is taken into account, the length of the linear dipoles that form the array must be taken into consideration. In this case, the radiated field expressions must be computed for the near field region (they can not be analyzed in the far field region). The field radiated by each element contributes not only to the radiated beam, but also to the distortion of the current distributions in the neighbor elements [33].

To calculate the linear array spatial correlation coefficients, subject to the coupling effect, it is necessary to establish the relation between the voltage vector $\boldsymbol{V}$, obtained at the array dipoles, and the signal sample vector $\boldsymbol{S}$, without coupling [34]. This relation is [8]

$$\boldsymbol{V} = \boldsymbol{Z}^{-1}\boldsymbol{S}, \tag{31}$$

in which

$$\boldsymbol{Z} = \begin{bmatrix} 1+\frac{Z_{11}}{Z_c} & \frac{Z_{12}}{Z_c} & \frac{Z_{13}}{Z_c} & \cdots & \frac{Z_{1L}}{Z_c} \\ \frac{Z_{21}}{Z_c} & 1+\frac{Z_{22}}{Z_c} & \frac{Z_{23}}{Z_c} & \cdots & \frac{Z_{2L}}{Z_c} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{L1}}{Z_c} & \frac{Z_{L2}}{Z_c} & \frac{Z_{L3}}{Z_c} & \cdots & 1+\frac{Z_{LL}}{Z_c} \end{bmatrix} \tag{32}$$

and

$$\boldsymbol{S} = \begin{bmatrix} e^{j0\beta d \sin\varphi} \\ e^{j1\beta d \sin\varphi} \\ \vdots \\ e^{j(L-1)\beta d \sin\varphi} \end{bmatrix}. \tag{33}$$

It is important to point out that the definition of the mutual impedance matrix in Equation 32 is accurate only for transmitting antennas. For receiving antenna arrays, such as the ones considered for DoA application in this paper, the mutual coupling effect characterized by the matrix in Equation 32 is not accurate because the mutual impedance elements are calculated based on the current distributions of transmitting antenna elements with excitation at the antenna ports. A more accurate modeling of the antenna mutual impedances, the so-called "receiving mutual impedance", can be found in [35]–[37].

However, Equation 32 can be used if the receiving antenna arrays for DoA application are excited by electromagnetic waves coming from a short distance (*e.g.*, indoor transmissions), then the current distribution is different for each antenna element.

In the matrix $L\times L$ in Equation 32, the elements Z_{mm} represent the self-impedance of the m-th dipole, Z_{mn} represents the mutual impedance between the m-th dipole and the n-th dipole and Z_c represents the load impedance of the antenna array dipoles [38]. Considering the elements a_{mn} of the inverse impedance matrix $\boldsymbol{Z}^{-1}$, the vector of voltages captured in the elements of the array can be written as

$$\boldsymbol{V} = \begin{bmatrix} \sum_{i=1}^{L} a_{1i} e^{j(i-1)\beta d \sin\varphi} \\ \sum_{i=1}^{L} a_{2i} e^{j(i-1)\beta d \sin\varphi} \\ \vdots \\ \sum_{i=1}^{L} a_{Li} e^{j(i-1)\beta d \sin\varphi} \end{bmatrix}. \tag{34}$$

If one takes two elements, denoted by $r_m(\varphi)$ and $r_n(\varphi)$, of the voltage vector $\boldsymbol{V}$, the square of the correlation coefficient modulus can be written as

$$|\rho_{mn}|^2 = \frac{1}{P_m P_n}\int r_m(\varphi) r_n^*(\varphi) p_\varphi(\varphi) d\varphi, \tag{35}$$

in which the average power P_m of the signal at the m-th array element was calculated in [17] and can be written as

$$\begin{aligned} P_m =& 2\sum_{k=1}^{L}\sum_{\substack{l=1\\ l>k}}^{L} \Re\{a_{mk}a_{ml}^*\} b_{kl} + \sum_{k=1}^{L}|a_{mk}|^2 \\ &+ 2\sum_{k=1}^{L}\sum_{\substack{l=1\\ l>k}}^{L} \Im\{a_{mk}a_{ml}^*\}\beta_{kl}. \end{aligned} \tag{36}$$

The integral $\int r_k(\varphi) r_l^*(\varphi) p(\phi) d\phi$ was also calculated in [17] and can be written as

$$\int r_k(\varphi) r_l^*(\varphi) p_\varphi(\varphi) d\varphi = \sum_{n=1}^{L}\sum_{m=1}^{L} a_{kn}a_{lm}^*(b_{nm}+j\beta_{nm}), \tag{37}$$

in which $b_{nm}+j\beta_{nm}$ is the complex spatial correlation coefficient calculated without coupling. Hence, the spatial correlation coefficients can be calculated by the expression

$$|\rho_{nm}|^2 = \frac{1}{P_n P_m}\left|\sum_{k=1}^{L}\sum_{l=1}^{L} a_{nk}a_{ml}^*\left(b_{kl}+j\beta_{kl}\right)\right|^2. \tag{38}$$

For the case of linear dipole elements of length l, with $l = n\lambda/2$, $n = 1, 3, 5, ...,$, aligned side by side and centrally fed, the real and imaginary parts of the mutual impedance between two dipoles, referred to as Dipole 1 and Dipole 2, can be written as [38]

$$\begin{aligned} R_{12} &= \frac{\eta_0}{4\pi}[2\text{Ci}(u_0) - \text{Ci}(u_1) - \text{Ci}(u_2)], \\ X_{12} &= -\frac{\eta_0}{4\pi}[2\text{Si}(u_0) - \text{Si}(u_1) - \text{Si}(u_2)], \end{aligned} \tag{39}$$

in which $u_0 = \beta d$, $u_1 = \beta(\sqrt{d^2+l^2}+l)$ and $u_2 = \beta(\sqrt{d^2+l^2}-l)$.

For the expressions, $\beta = 2\pi/\lambda$ is the wave number, η_0 is the medium impedance, approximately 120π Ohms, and $\text{Ci}(x)$ and $\text{Si}(x)$ are respectively the integral sine and cosine functions. Thus, the elements Z_{mn} of the matrix $\boldsymbol{Z}$ are given by $Z_{mn} = R_{mn} + jX_{mn}$, in which the distance d_{mn} between the elements m and n is given by

$$d_{mn} = d|m-n|, \tag{40}$$

for the linear array.

V. Results

For the numerical evaluation of the expressions presented in this paper, the elements of the linear array are magnetically coupled.

In Figure 1, the SEP curves are presented, considering both Gaussian distribution, for $\sigma_\varphi = 27^o$ and $\varphi_o = 48.5^o$, and Von Mises distribution, for $\kappa = 4.7$ and $\varphi_o = 48.5^o$. The remaining parameters are $m = 0.8$, $d = 0.25\lambda$, $l = 0.6\lambda$, $M = 64$ and $\theta = 60^o$.

The Gaussian model was originally proposed in [39] to characterize the azimuthal direction of arrival of electromagnetic waves at a linear antenna array on a base station. In this model, the waves radiated by the mobile station are spread and reflected by obstacles of the urban environment. The reflected components that can reach the base station antenna array have a power spectrum which is concentrated on a mean direction φ_o and fades on a interval $(-\pi+\varphi_o, \pi+\varphi_o)$. This model was also applied to characterize the analysis of a unified model of channel for mobile radio systems with linear antenna arrays by [40], where typical values of σ_{varphi} range from 5^o to 60^o. Meanwhile, typical values of φ_o range from 0^o to 180^o.

The spacing between array elements is generally chosen as a half-wavelength. However, this spacing may be smaller in more compact array arrangements, producing an increase in correlation and magnetic coupling. The parameter m depends on the urban environment. Multipath propagation tend to make the fading behave as Rayleigh. In this case, typical values for m can be chosen between 0.8 and 2.0.

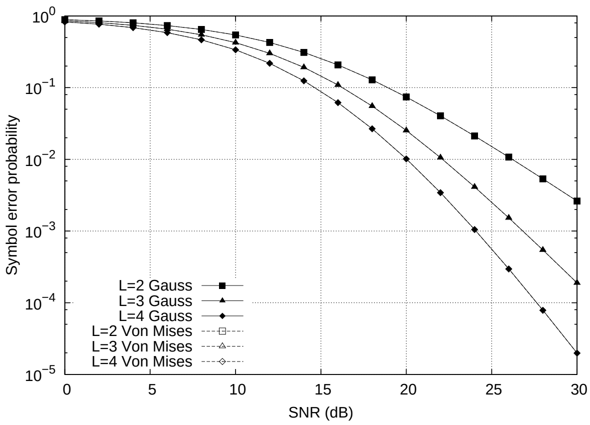


Fig. 1. SEP for θ-QAM scheme under correlated Nakagami-m fading and MRC receiver with L branches, for $\theta = 60^o$, $M = 64$, $l = 0.6\lambda$, $d = 0.25\lambda$ and different values of L.

The curves in Figure 1 are superimposed since the pdfs of the Von Mises and Gaussian distributions present the same mean and almost the same variance. In the Gaussian pdf, the variance is given by $1/\kappa = 1/4.7 \quad \text{rad} = 12.19^o$. In the Von Mises pdf, the variance is given by $\sigma_\varphi^2 = (27 \cdot \pi/180)^2$, which is equivalent to the angle of 0.222 rad $= 12.72^o$. The difference from the variance $1/\kappa$ is only 0.53^o. It can be noted in Figure 1 that the addition of one branch in the MRC receiver leads to a diversity gain up to 7 dB considering the symbol error probability of 10^{-3}.

In Figure 2, SEP curves are presented as a function of the DoA for the Von Mises distribution and two values of m. It is observed that the diversity effect is more effective when the fading is smooth, for $m = 1.5$. For $m = 0.8$, the Nakagami fading is more severe than the Rayleigh fading, $\kappa = 4.7$, and, in this case, the signal components arrive the MRC receiver with an angle variation smaller than the one for $\kappa = 3$. This narrowing in the angle region around the main direction of arrival φ_o contributes for enhancing both spatial correlation and magnetic coupling effect between the elements, directly influencing the SEP.

In Figure 3, SEP curves are presented as a function of the separation distance between the array elements, considering the Gaussian distribution for the angle of arrival.

It is observed in Figures 2 and 3 that the direction of arrival φ_o has a stronger impact on the SEP, when compared to the

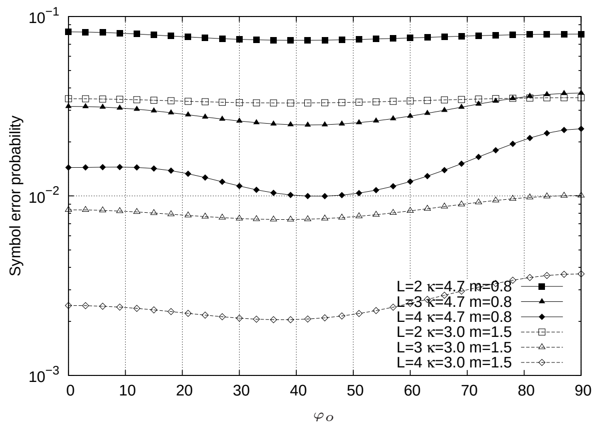


Fig. 2. SEP for θ-QAM scheme as a function of the direction of arrival φ_o, under correlated Nakagami-m fading and MRC receiver with L branches, for $\theta = 60^o$, $M = 64$, $l = 0.6\lambda$, $d = 0.25\lambda$, $SNR = 20$ dB and different values of L.

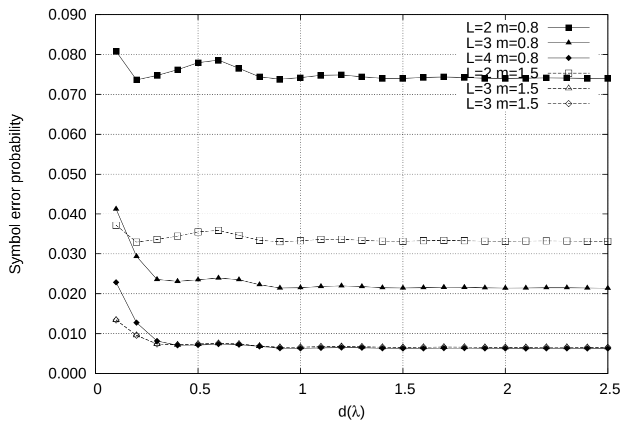
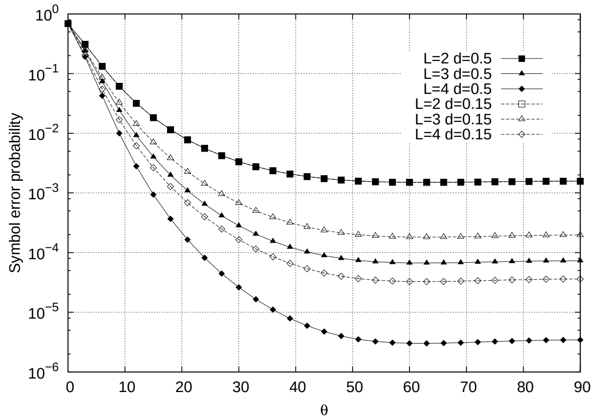


Fig. 3. SEP for θ-QAM scheme as a function of the separation distance between antenna array elements, under correlated Nakagami-m fading and MRC receiver with L branches, for $\varphi_o = 48^o$, $\sigma_\varphi = 27^o$, $\theta = 60^o$, $M = 64$, $l = 0.6\lambda$, $SNR = 20$ dB and different values of L.

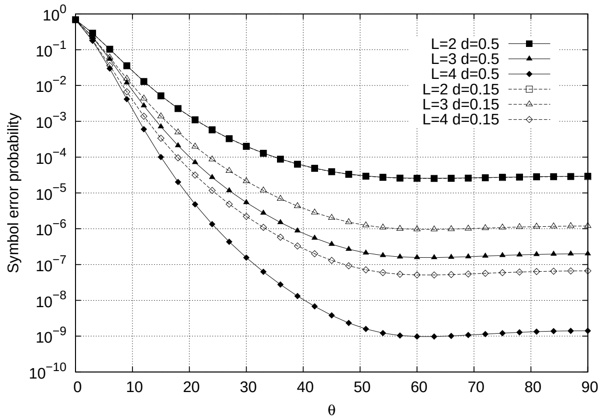
separation distance between the array elements, mainly for $d > 0.5\lambda$. This is due to the fact that the spatial correlation and the coupling between the elements are more impacted by the DoA. The distance d has a deeper impact for values much smaller than $\lambda/4$.

In Figure 4, SEP curves for $M = 16$ and $M = 64$ are presented as a function of the angle θ of the constellation θ-QAM, for a Gaussian distribution for the DoA and two values of d (as a function of λ), for a linear array. It is observed from Figure 4 that the impact of the diversity order of the MRC receiver in the SEP is greater than the one of the constellation angle θ-QAM. It is also observed that the smaller SEP values are obtained for $\theta = 60^o$ and for the case of less severe fading ($m = 2.0$).

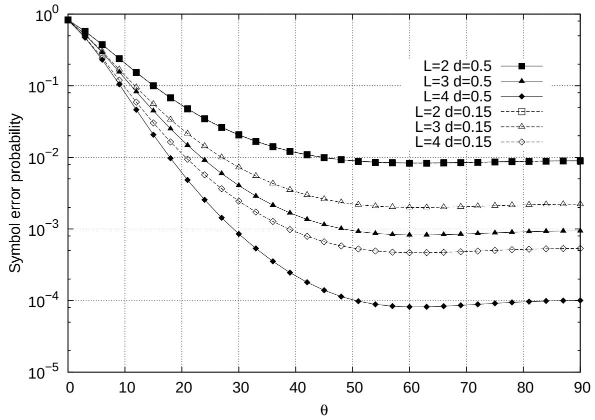
In Figure 5, SEP curves are presented as a function of the SNR for different values of distance between the elements of the linear array, fading intensity and length of the array elements. It is observed that for severe fading ($m = 0.6$) and strong coupling between the elements ($d = 0.15\lambda$ and $L = 0.8\lambda$), more than four branches are needed to maintain the



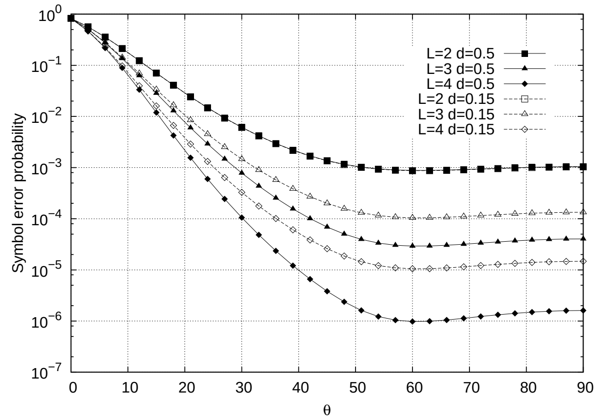
(a) $m = 0.8$, M=16.



(b) $m = 1.6$, M=16.



(c) $m = 1.0$, M=64.



(d) $m = 2.0$, M=64.

Fig. 4. SEP for θ-QAM scheme as a function of the angle θ of the constellation θ-QAM, under correlated Nakagami-m fading and MRC receiver with L branches, for $\varphi_o = 45^o$, $\sigma_\varphi = 40^o$, $l = 0.6\lambda$, $SNR = 25$ dB and different values of L.

SEP below 10^{-4}.

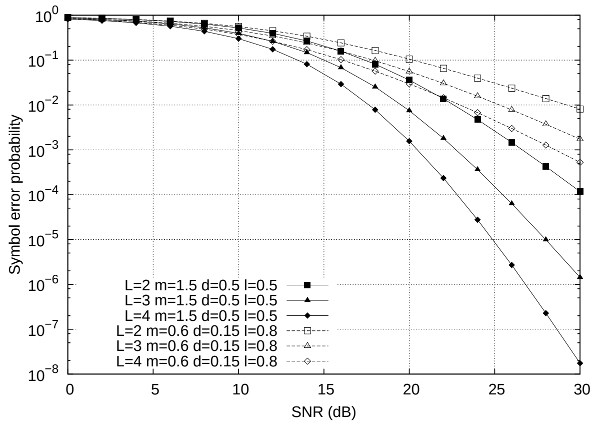


Fig. 5. SEP for θ-QAM scheme as a function of the SNR, under correlated Nakagami-m fading and MRC receiver with L branches, for $\sigma_\varphi = 40^o$, $\varphi_o = 60^o$, $M = 64$, $\theta = 60^o$ and different values of L.

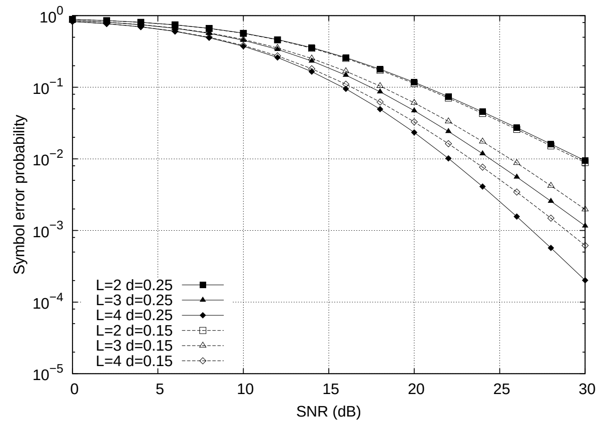


Fig. 6. SEP for θ-QAM scheme as a function of the SNR, under correlated Nakagami-m fading and MRC receiver with L branches, for $\varphi_o = 60^o$, $\kappa = 2.0$, $M = 64$, $l = 0.8\lambda$, $\theta = 45^o$, $m = 0.6$ and different values of L.

Figure 6 shows SEP curves as a function of the SNR, for different values of distance between the elements of the linear array, considering the use of the Von Mises distribution.

It is observed in Figure 6 that a decrease in the distance between the array elements, considering the dipole length $l = 0.8\lambda$, has an impact in the SEP of receivers with four branches that is greater than in the SEP of receivers with two branches. This is due to the fact that the arrays with four elements suffer more coupling from neighbor elements than arrays with three or two elements.

VI. Conclusion

In this article the symbol error probability (SEP) of the θ-QAM scheme under correlated Nakagami fading is evaluated using Von Mises and Gaussian distributions to model the direction of arrival (DoA) of the electromagnetic waves for antenna arrays. An expression for the SEP is presented in terms of the characteristic function of the signal to noise ratio (SNR) at the output of the receiver, taking into account the effect of the magnetic coupling between the elements of the array. An important aspect of the Von Mises Distribution is the fact that it models more adequately anisotropic propagation processes, when compared to the uniform distribution, and leads to mathematical expressions for the spatial correlation coefficients that are simpler than the ones obtained with Gaussian and cosine distributions. It is observed that the DoA of the electromagnetic waves in the array, with respect to the MRC receiver, has a stronger influence on the SEP than the separation distance between the array elements. Additionally, the number of branches in the receiver has a stronger influence against the effects of coupling than the angle θ of the constellation θ-QAM.

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