Closed-Form Receiver for Multi-Hop MIMO Relay Systems with Tensor Space-Time Coding

Danilo S. Rocha, Gérard Favier and C. Alexandre R. Fernandes

***Abstract*—A multi-hop multi input multi output (MIMO) relay system with tensor coding at the source and relays is proposed in this paper. The tensor of signals received at destination satisfies a new tensor model, called high-order nested Tucker decomposition (HONTD), which generalizes an existing tensor decomposition. Exploiting this HONTD model allows to derive a closed-form semi-blind receiver for jointly estimating the information symbols and the individual channels. Identifiability conditions and ambiguity relations are derived. Monte Carlo simulation results are provided to evaluate the performance of the proposed receiver in terms of channels and symbols estimation, and the impact of the number of relays is illustrated.**

***Index Terms*—Closed-form receiver, multi-hop MIMO system, nested Tucker decomposition, tensor coding, tensor models.**

I. Introduction

DURING the last decade, tensor models have been extensively used for designing MIMO point-to-point wireless communication systems as well as cooperative networks [1]. The main motivation for using tensor-based approaches is related to their natural capability to model multimodal signals, with useful uniqueness properties under mild conditions. In wireless communications, tensor tools are very useful for designing tensor coding and semi-blind receivers [2]-[15].

In MIMO relaying systems, the effectiveness of exploiting available diversities depends on the accuracy of channel state information (CSI) of each hop. Some works propose tensor-based supervised channel estimation, with the drawback to be bandwidth consuming [4]-[7], whereas others present semi-blind receivers for channels and symbols estimation [8]-[12], [15]. In the context of multi-hop relaying systems using tensor approaches, one can mention the works [7], [8], [12]-[15].

In this paper, we propose a new tensor model for a multi-hop MIMO relaying system composed of K relays operating with tensor space-time coding (TSTC) [2] and the amplify-and-forward (AF) protocol. This system can be viewed as a generalization of recently proposed systems [9]-[12], [15] to the multi-hop case using tensor coding, leading to the description of a high-order tensor model in a compact way. The present paper extends previous works in different ways, either by using a more general relay coding, by extending these works to the multihop case and/or by using a different estimation algorithm. Assuming a third-order TSTC at the source and the relays, we show that the signals received at destination satisfy a new high-order nested Tucker decomposition (HONTD). This model results from the contraction of several Tucker models in a train format, generalizing the nested Tucker decomposition (NTD) introduced in [10] to a $(K+3)$-th order tensor, for $K \geq 2$.

Considering the tensor codings known at the destination, we derive a generic closed-form semi-blind receiver based on a recurrent algorithm that takes into account least squares (LS) estimates of Kronecker products (KP), denoted by LSKP, for jointly estimating the symbols and the individual channels. The LSKP receiver exploits matrix unfoldings of coding tensors under the form of unitary matrices, avoiding noise enhancement and yielding better performance when compared to random coding. Monte Carlo simulations are provided to illustrate the performance of the receiver, showing the great performance of the proposed multi-hop receiver. The main original contributions of the present work can be summarized as follows: (i) presentation of a new tensor model, called HONTD, which generalizes the existing NTD [10]; (ii) presentation of a new multi-hop MIMO relaying system with TSTC at all the nodes, generalizing recently proposed systems [9]-[12], [15]; (iii) proposition of a closed-form recurrent semi-blind receiver for the considered system model.

Notation: scalars, column vectors, matrices and tensors of order higher than two are denoted by lower-case, boldface lower-case, boldface upper-case, and calligraphic letters (a, $\mathbf{a}$, $\mathbf{A}$ and $\mathcal{A}$, respectively). $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ denote the transpose and the Moore-Penrose pseudo-inverse of $\mathbf{A}$. The Kronecker product is denoted by $\otimes$. Given a N-th order tensor $\mathcal{X} \in \mathbb{C}^{I_1\times\cdots\times I_N}$, the third-order tensor $\mathcal{X}_{J_1\cdots J_{N-2}\times J_{N-1}\times J_N}$ is a contracted form of $\mathcal{X}$ obtained by combining $N-2$ modes, where $\{J_1,\cdots,J_N\}$ is any permutation of $\{I_1,\cdots,I_N\}$. The matrix $\mathbf{X}_{J_1\cdots J_{N-1}\times J_N}$ is a tall unfolding of $\mathcal{X}$ whose the entries are $x_{j_1,\cdots,j_N} = \left[X_{J_1\cdots J_{N-1}\times J_N}\right]_{(j_1-1)J_2\cdots J_{N-1}+\cdots+(j_{N-2}-1)J_{N-1}+j_{N-1},j_N}$.

The mode-n product of $\mathcal{X}$ with $\mathbf{U}\in\mathbb{C}^{P_n\times I_n}$, denoted by $\mathcal{A} = \mathcal{X}\times_n \mathbf{U} \in \mathbb{C}^{I_1\times\cdots\times I_{n-1}\times P_n\times I_{n+1}\times\cdots\times I_N}$, is given by

$$a_{i_1,\cdots,i_{n-1},p_n,i_{n+1},\cdots,i_N} = \sum_{i_n=1}^{I_n} x_{i_1,\cdots,i_n,\cdots,i_N} u_{p_n,i_n}. \quad (1)$$

Let $\mathcal{Y} \in \mathbb{C}^{J_1\times\cdots\times J_M}$ be a tensor such that the dimension of its first mode is equal to the dimension of the last mode of $\mathcal{X}$ $(I_N = J_1)$. The contraction of $\mathcal{X}$ with $\mathcal{Y}$, denoted by $\mathcal{B} = \mathcal{X}\underline{\times}_N\mathcal{Y} \in \mathbb{C}^{I_1\times\cdots\times I_{N-1}\times J_2\times\cdots\times J_M}$, is given by [16]

$$b_{i_1,\cdots,i_{N-1},j_2,\cdots,j_M} = \sum_{i_N=1}^{I_N} x_{i_1,\cdots,i_N} y_{i_N,j_2,\cdots,j_M}. \quad (2)$$

II. System Model

Let us consider the multi-hop MIMO relaying system illustrated in Fig. 1, composed of a source (S), K relays

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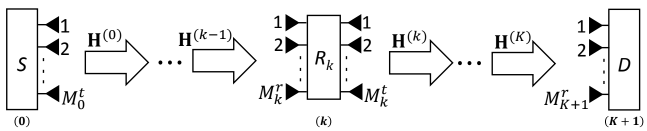

Fig. 1. Multi-hop MIMO relay system.

$(R_1, ..., R_K)$ and a destination (D) node. The numbers of transmit and receive antennas at node k are denoted by M_k^t and M_k^r, respectively, with $k \in \{0, ..., K+1\}$. The nodes indexed by 0 and $K+1$ correspond to the source and the destination, respectively. The transmission consists of $K+1$ steps via K relays. The source and the relays encode the signals to be transmitted by means of a TSTC, and the relays operate in half-duplex mode, using the AF protocol, i.e., retransmitting the received signals without decoding. Synchronization is assumed at the symbol level and the channels undergo frequency-flat fading, $\mathbf{H}^{(k)} \in \mathbb{C}^{M_{k+1}^r \times M_k^t}$ being the channel matrix between the nodes k and $k+1$, for $k = 0, ..., K$.

The symbol matrix encoded by the source is denoted by $\mathbf{S} \in \mathbb{C}^{N \times R}$, R being the number of data streams transmitted during each symbol period and N being the number of data symbols per data stream. The coding tensor $\mathcal{C}^{(0)} \in \mathbb{C}^{M_0^t \times P_0 \times R}$ gives the coded signals $\mathcal{Y}^{(0)} = \mathcal{C}^{(0)} \times_3 \mathbf{S} \in \mathbb{C}^{M_0^t \times P_0 \times N}$, where P_0 is the time spreading length of TSTC. After transmission through the channel $\mathbf{H}^{(0)}$, R_1 receives a signal tensor that satisfies the following third-order Tucker model

$$\begin{aligned}\tilde{\mathcal{X}}^{(1)} &= \mathcal{X}^{(1)} + \mathcal{N}^{(1)} = \mathcal{Y}^{(0)} \times_1 \mathbf{H}^{(0)} + \mathcal{N}^{(1)} \\ &= \mathcal{C}^{(0)} \times_1 \mathbf{H}^{(0)} \times_3 \mathbf{S} + \mathcal{N}^{(1)} \in \mathbb{C}^{M_1^r \times P_0 \times N}, \quad (3)\end{aligned}$$

where $\mathcal{N}^{(1)}$ is the additive white Gaussian noise (AWGN) tensor at the relay R_1.

Each relay R_k re-encodes the received signals by means of the coding tensor $\mathcal{C}^{(k)} \in \mathbb{C}^{M_k^t \times P_k \times M_k^r}$, for $k = 1, ..., K$, resulting in the coded signals $\mathcal{Y}^{(k)} = \mathcal{C}^{(k)} \underline{\times}_3 \tilde{\mathcal{X}}^{(k)} \in \mathbb{C}^{M_k^t \times P_k \times \cdots \times P_0 \times N}$ to be transmitted. After transmission via channel $\mathbf{H}^{(k)}$, the signals received at the node $k+1$ form a $(k+3)$-th order tensor given by

$$\begin{aligned}\tilde{\mathcal{X}}^{(k+1)} &= \mathcal{Y}^{(k)} \times_1 \mathbf{H}^{(k)} + \mathcal{N}^{(k+1)} \quad (4) \\ &= \mathcal{T}^{(k)} \underline{\times}_3 \tilde{\mathcal{X}}^{(k)} + \mathcal{N}^{(k+1)} \in \mathbb{C}^{M_{k+1}^r \times P_k \times \cdots \times P_0 \times N},\end{aligned}$$

where $\mathcal{N}^{(k+1)}$ is the AWGN tensor at the node $k+1$ and $\mathcal{T}^{(k)}$ is defined as a Tucker-$(1,3)$ decomposition

$$\mathcal{T}^{(k)} = \mathcal{C}^{(k)} \times_1 \mathbf{H}^{(k)} \in \mathbb{C}^{M_{k+1}^r \times P_k \times M_k^r}. \quad (5)$$

From the recurrent relation (4), we deduce

$$\begin{aligned}\tilde{\mathcal{X}}^{(k+1)} &= \mathcal{X}^{(k+1)} + \bar{\mathcal{N}}^{(k+1)}, \quad (6) \\ \mathcal{X}^{(k+1)} &= \mathcal{T}^{(k)} \underline{\times}_3 \mathcal{X}^{(k)}, \quad (7) \\ \bar{\mathcal{N}}^{(k+1)} &= \mathcal{N}^{(k+1)} + \mathcal{T}^{(k)} \underline{\times}_3 \bar{\mathcal{N}}^{(k)}. \quad (8)\end{aligned}$$

The noiseless received signals tensor $\mathcal{X}^{(k+1)}$ can be written in the following Tucker train format

$$\mathcal{X}^{(k+1)} = \mathcal{T}^{(k)} \underline{\times}_3 \mathcal{T}^{(k-1)} \underline{\times}_3 \cdots \underline{\times}_3 \mathcal{T}^{(1)} \underline{\times}_3 \mathcal{X}^{(1)}, \quad (9)$$

$\mathcal{X}^{(1)}$ being defined in (3). This tensor model generalizes the NTD [10] to the order $(K+3)$, where $K \geq 1$ is the number of relays, yielding the new HONTD model.

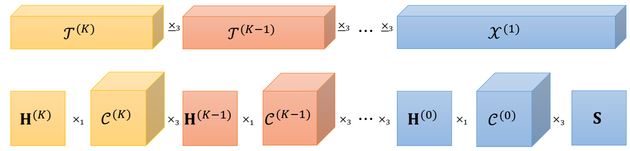

Fig. 2. High-order nested Tucker decomposition of a tensor of order $K+3$.

Fig. 2 represents a block-diagram of the HONTD-based system which can be viewed as a nesting of Tucker-$(2,3)$ models $(\mathcal{C}^{(k)}, \mathbf{H}^{(k)}, \mathbf{H}^{(k-1)})$, where two successive models (for k and $k+1$) share the factor matrix $\mathbf{H}^{(k)}$. At the top level of Fig. 2, the HONTD is represented as the contraction of the Tucker-$(1,3)$ models $\mathcal{T}^{(k)}$ defined in (5), via $\underline{\times}_3$ tensor products, illustrating (9).

III. LSKP RECEIVER

In this section, we propose a closed-form semi-blind receiver to estimate the symbol and channel matrices of the multi-hop MIMO relay system presented in Section II. The coding tensors $\mathcal{C}^{(k)}$ used by the source and the relays are assumed known at destination. In the sequel, for the sake of simplicity, the equations are derived in the noiseless case.

By combining the last $k+1$ modes of $\mathcal{X}^{(k+1)}$, given in (7), and using the definition (5), we obtain a contracted form of $\mathcal{X}^{(k+1)}$ that satisfies the following Tucker-$(2,3)$ model

$$\begin{aligned}\mathcal{X}^{(k+1)}_{M_{k+1}^r \times P_k \times P_{k-1} \cdots P_0 N} &= \mathcal{T}^{(k)} \times_3 \mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r} \quad (10) \\ &= \mathcal{C}^{(k)} \times_1 \mathbf{H}^{(k)} \times_3 \mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r},\end{aligned}$$

where $\mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r}$ is a tall matrix unfolding of $\mathcal{X}^{(k)}$. A tall mode-2 unfolding of (10) is given by

$$\mathbf{X}^{(k+1)}_{M_{k+1}^r P_{k-1} \cdots P_0 N \times P_k} = \left(\mathbf{H}^{(k)} \otimes \mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r}\right) \mathbf{C}^{(k)}_{M_k^t M_k^r \times P_k}. \quad (11)$$

Let us define the KP $\mathbf{\Omega}^{(k)} = \mathbf{H}^{(k)} \otimes \mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r} \in \mathbb{C}^{M_{k+1}^r P_{k-1} \cdots P_0 N \times M_k^t M_k^r}$, for $k = 1, ..., K$, such that we derive the LS estimate of $\mathbf{\Omega}^{(k)}$ as

$$\widehat{\mathbf{\Omega}}^{(k)} = \mathbf{X}^{(k+1)}_{M_{k+1}^r P_{k-1} \cdots P_0 N \times P_k} \left(\mathbf{C}^{(k)}_{M_k^t M_k^r \times P_k}\right)^{\dagger}. \quad (12)$$

The main idea of the proposed receiver is to estimate recursively the channel matrix $\mathbf{H}^{(k)}$ and the matrix unfolding $\mathbf{X}^{(k)}_{P_{k-1} \cdots P_0 N \times M_k^r}$, from $k = K$ to $k = 1$, by using the LS estimate (12) of $\mathbf{\Omega}^{(k)}$ and applying a singular-value decomposition (SVD)-based low-rank approximation algorithm [17].

In order to avoid error propagation effects, the symbol matrix $\mathbf{S}$ is directly estimated using the tensor of signals received at destination, as detailed below. From (9), we have $\mathcal{X}^{(K+1)} = \mathcal{A} \underline{\times}_3 \mathcal{X}^{(1)}$, where $\mathcal{A}$ is an auxiliary tensor given by

$$\mathcal{A} = \mathcal{T}^{(K)} \underline{\times}_3 \cdots \underline{\times}_3 \mathcal{T}^{(1)} \in \mathbb{C}^{M_{K+1}^r \times P_K \times \cdots \times P_1 \times M_1^r}. \quad (13)$$

By combining the first $K+1$ modes of $\mathcal{A}$, the tensor $\mathcal{X}^{(K+1)}$ can be rewritten as

$$\mathcal{X}^{(K+1)}_{M_{K+1}^r P_K \cdots P_1 \times P_0 \times N} = \mathcal{X}^{(1)} \times_1 \mathbf{A}_{M_{K+1}^r P_K \cdots P_1 \times M_1^r}. \quad (14)$$

By replacing (3) into (14), we get the following Tucker model

$$\mathcal{X}^{(K+1)}_{M^r_{K+1}P_K\cdots P_1\times P_0\times N} = \mathcal{C}^{(0)} \times_1 \mathbf{B} \times_3 \mathbf{S}, \tag{15}$$

where $\mathbf{B} = \mathbf{A}_{M^r_{K+1}P_K\cdots P_1\times M^r_1}\mathbf{H}^{(0)} \in \mathbb{C}^{M^r_{K+1}P_K\cdots P_1\times M^t_0}$. By considering a tall mode-2 unfolding of (15), we obtain

$$\mathbf{X}^{(K+1)}_{M^r_{K+1}P_K\cdots P_1 N\times P_0} = (\mathbf{B}\otimes\mathbf{S})\,\mathbf{C}^{(0)}_{M^t_0 R\times P_0}. \tag{16}$$

The LS estimate of $\mathbf{\Omega}^{(0)} = \mathbf{B}\otimes\mathbf{S}$, is given by

$$\widehat{\mathbf{\Omega}}^{(0)} = \mathbf{X}^{(K+1)}_{M^r_{K+1}P_K\cdots P_1 N\times P_0}\left(\mathbf{C}^{(0)}_{M^t_0 R\times P_0}\right)^{\dagger}. \tag{17}$$

Once $\widehat{\mathbf{\Omega}}^{(0)}$ estimated using the signals received at destination, the SVD-based low-rank approximation algorithm allows us to estimate the Kronecker factors $\mathbf{B}$ and $\mathbf{S}$ directly from the signals received at destination using (17). The LS estimate of the channel matrix $\mathbf{H}^{(0)}$ is then given by

$$\widehat{\mathbf{H}}^{(0)} = \left(\widehat{\mathbf{A}}_{M^r_{K+1}P_K\cdots P_1\times M^r_1}\right)^{\dagger}\widehat{\mathbf{B}}, \tag{18}$$

where $\widehat{\mathbf{A}}_{M^r_{K+1}P_K\cdots P_1\times M^r_1}$ is obtained using (13), with the channel matrices replaced by their estimates $\widehat{\mathbf{H}}^{(k)}$, for $k = 1,...,K$, obtained in previous steps of the algorithm.

Identifiability conditions and ambiguity relations

The parameter identifiability is linked to the uniqueness of the LS estimates (12) and (17), which require the unfoldings $\mathbf{C}^{(k)}_{M^t_k M^r_k\times P_k}$ and $\mathbf{C}^{(0)}_{M^t_0 R\times P_0}$ to be full row rank for uniqueness of their right inverse. That leads to the necessary identifiability conditions $P_k \geq M^t_k M^r_k$ and $P_0 \geq M^t_0 R$. Regarding (18), the unfolding $\widehat{\mathbf{A}}_{M^r_{K+1}P_K\cdots P_1\times M^r_1}$ must be left-invertible, which leads to the condition $M^r_{K+1}P_K\cdots P_1 \geq M^r_1$.

Moreover, the factors of a KP can only be estimated up to scalar ambiguities, as shown in [17]. As the steps of the proposed receiver perform the factorization of the KP matrix $\mathbf{\Omega}^{(k)}$, the estimated factors have the ambiguities $\widehat{\mathbf{H}}^{(k)} = \delta_{\mathbf{H}^{(k)}}\mathbf{H}^{(k)}$ and $\widehat{\mathbf{S}} = \delta_{\mathbf{S}}\,\mathbf{S}$, for $k = 0,...,K$. By comparison, the receiver of [12], based on the estimation of the factors of Khatri-Rao products, needs the knowledge of one row of each factor matrix.

The scaling ambiguity of $\widehat{\mathbf{S}}$ can be removed by assuming the a priori knowledge of one pilot symbol ($s_{1,1}$). Concerning the scaling ambiguity of $\widehat{\mathbf{H}}^{(k)}$, in order to plot the simulation results, we assumed that one coefficient ($h^{(k)}_{1,1}$) of each channel $\widehat{\mathbf{H}}^{(k)}$ is known at destination. This assumption has already been adopted in other works [4], [7], [9]-[12], [15] in the context of relaying systems. In practice, such a priori information could be obtained by a simple LS estimation using a pilot-symbol generated by the relays [9]. If the use of a pilot-symbol isn't possible, the channel matrices $\widehat{\mathbf{H}}^{(k)}$ are estimated up to a scalar constant. However, this ambiguity would not affect the symbol estimation neither the design of precoding schemes with channel state information (CSI). The ambiguities are then cancelled as follows $\widehat{\mathbf{S}} \leftarrow (\delta_{\mathbf{S}})^{-1}\,\widehat{\mathbf{S}}$ and $\widehat{\mathbf{H}}^{(k)} \leftarrow (\delta_{\mathbf{H}^{(k)}})^{-1}\,\widehat{\mathbf{H}}^{(k)}$, with $\delta_{\mathbf{S}} = \widehat{s}_{1,1}/s_{1,1}$ and $\delta_{\mathbf{H}^{(k)}} = \widehat{h}^{(k)}_{1,1}/h^{(k)}_{1,1}$.

The LSKP receiver is summarized in Algorithm 1. Assuming $M^t_k = M^r_k = M = R$ and $P_k = P$ for $k = 1,...,K$, the computational complexity of the proposed receiver is $\mathcal{O}(M^3P^K[NP + M^{K-2}])$.

Algorithm 1 LSKP receiver

Stage 1: estimation of $\mathbf{H}^{(k)}$, for $k = 1,\ldots,K$

1. $\widehat{\mathbf{X}}^{(K+1)}_{M^r_{K+1}P_{K-1}\cdots P_0 N\times P_K} = \tilde{\mathbf{X}}^{(K+1)}_{M^r_{K+1}P_{K-1}\cdots P_0 N\times P_K}$.
2. Calculate $\widehat{\mathbf{H}}^{(k)}$ and $\widehat{\mathbf{X}}^{(k)}_{P_{k-1}\cdots P_0 N\times M^r_k}$ from $\widehat{\mathbf{\Omega}}^{(k)}$:

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for k = K : 1
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$\widehat{\mathbf{\Omega}}^{(k)} = \widehat{\mathbf{X}}^{(k+1)}_{M^r_{k+1}P_{k-1}\cdots P_0 N\times P_k}\left(\mathbf{C}^{(k)}_{M^t_k M^r_k\times P_k}\right)^{\dagger}$

Apply the low-rank approximation algorithm [12]

$\widehat{\mathbf{X}}^{(k)}_{M^r_k P_{k-2}\cdots P_0 N\times P_{k-1}} \leftarrow reshape\left(\widehat{\mathbf{X}}^{(k)}_{P_{k-1}\cdots P_0 N\times M^r_k}\right)$

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end
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Stage 2: estimation of $\mathbf{H}^{(0)}$ and $\mathbf{S}$

3. Calculate $\widehat{\mathbf{B}}$ and $\widehat{\mathbf{S}}$ from $\widehat{\mathbf{\Omega}}^{(0)}$:

$\widehat{\mathbf{\Omega}}^{(0)} = \mathbf{X}^{(K+1)}_{M^r_{K+1}P_K\cdots P_1 N\times P_0}\left(\mathbf{C}^{(0)}_{M^t_0 R\times P_0}\right)^{\dagger}$

Apply the low-rank approximation algorithm [12]

4. Build $\widehat{\mathcal{A}}$ from (13), with $\widehat{\mathcal{T}}^{(k)}$ composed by $\widehat{\mathbf{H}}^{(k)}$
5. Calculate $\widehat{\mathbf{H}}^{(0)}$ using $\widehat{\mathbf{H}}^{(0)} = \left(\widehat{\mathbf{A}}^{(0)}_{M^r_{K+1}P_K\cdots P_1\times M^r_1}\right)^{\dagger}\widehat{\mathbf{B}}$
6. Eliminate the scaling ambiguities and project the estimated symbols onto the symbol alphabet.

IV. Simulation Results

In this section, we provide simulation results to illustrate the efficiency of the proposed receiver. The results were averaged over at least 5×10^4 Monte Carlo runs. The symbol-error-rate (SER) and channel normalized mean square error (NMSE) are plotted as function of the transmission power to noise spectral density ratio (P_T/N_0). At each run, P_T is fixed and N_0 was calculated to provide the desired P_T/N_0 value. The transmitted symbols are 4-QAM modulated and we assume flat-fading channels, with independent and identically distributed complex Gaussian entries. The variance of the channel coefficients follows an exponential path-loss model given by $\sigma^2_{\mathbf{H}} = 1/d^4$, where $d = D/(K+1)$ is the distance between two relays, and D is the distance between the source and destination arbitrarily chosen equal to 1. The relays are uniformly distributed between the source and the destination.

The coding tensors $\mathcal{C}^{(k)}$ are normalized in such a way that all the nodes have the same transmission power and the total system power (arbitrarily chosen equal to 1) is kept constant, regardless of the number of relays and antennas [15]. Three different choices for the coding tensors are considered: i) tensors with elements of unit magnitude and phase randomly drawn from a uniform distribution between 0 and 2π; ii) tensors such that the unfolding $\mathbf{C}^{(0)}_{M^t_0 R\times P_0}$ (resp. $\mathbf{C}^{(k)}_{M^t_k M^r_k\times P_k}$) is a truncated discrete Fourier transform (DFT) matrix, i.e., composed of the first $M^t_0 R$ (resp. $M^t_k M^r_k$) rows of the DFT matrix of dimension $P_0\times P_0$ (resp. $P_k\times P_k$); iii) tensors such that the unfolding $\mathbf{C}^{(0)}_{M^t_0 R\times P_0}$ (resp. $\mathbf{C}^{(k)}_{M^t_k M^r_k\times P_k}$) is a truncated Hadamard matrix, i.e., composed of the first $M^t_0 R$ (resp. $M^t_k M^r_k$) rows of the Hadamard matrix of dimension $P_0\times P_0$ (resp. $P_k\times P_k$). The tensor codings ii) and iii) avoid the computation of the pseudo-inverses in (12) and (17), and prevent noise enhancement.

The parameters used in the simulations are indicated above each figure. Fig. 3 compares the SER versus P_T/N_0 obtained with the proposed LSKP receiver, for three different coding tensors and two relays ($K = 2$). We also show the SER

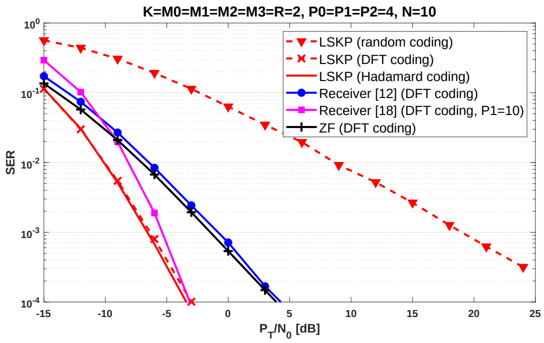

Fig. 3. SER performance comparison for $K = 2$.

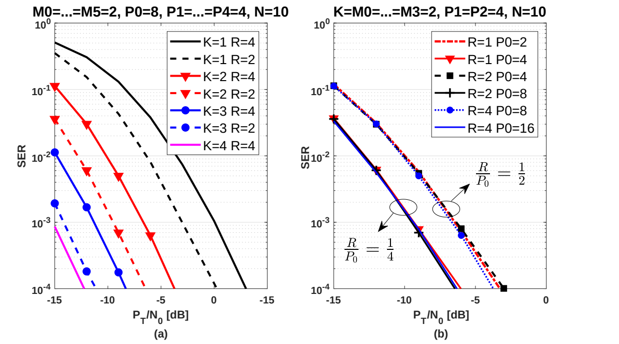

Fig. 4. SER performance with the LSKP receiver (a) for different numbers of relays and (b) for different values of R and P_0.

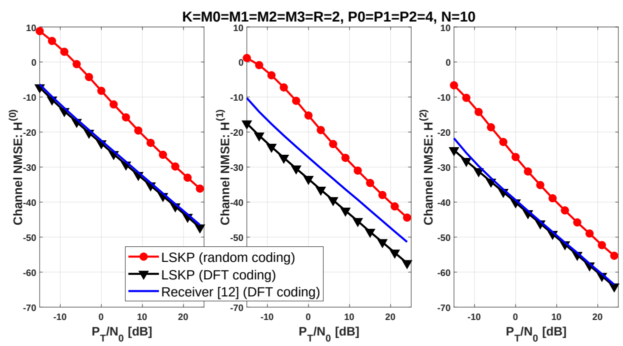

Fig. 5. NMSE of the individual channel estimates for $K = 2$.

performance of two existing tensor-based receivers. The first one is a receiver based on a Khatri-Rao factorization (KRF) algorithm proposed in [12] for a multi-hop relaying system that uses a simplified Khatri-Rao space-time coding. The second receiver is a LSKP technique proposed in [18] for a two-hop multirelay system with TSTC that use orthogonal relays to provide cooperative diversity. The parameters were chosen to ensure roughly the same transmission rate for all the systems. For the proposed system, the transmission rate is proportional to $R/P_0(1 + P_1 + P_1P_2)$, while for the systems of [12] and [18], it is proportional to $M_0^t/P_0(1 + P_1 + P_1P_2)$ and $R/P_0(P_1K + 1)$, respectively. We also plot the performance of the zero-forcing (ZF) receiver, given by: $\widehat{\mathbf{S}}^T = \left[(\mathbf{B} \otimes \mathbf{I}_{P_0})\,\mathbf{C}^{(0)}_{M_0^t P_0 \times R}\right]^\dagger \tilde{\mathbf{X}}^{(3)}_{M_3^T P_2 P_1 P_0 \times N}$.

In Fig. 3, the DFT and Hadamard codings with the LSKP receiver give significant SER improvements, overcoming the performance obtained with the other receivers. That comes from the fact that, in (12) and (17), the coding unfolded matrices are unitary, which avoids noise enhancement. It is worth mentioning that a coding with unitary unfoldings is feasible for the LSKP receiver, due to the use of only one matrix unfolding of each coding tensor. This orthogonality property can not be exploited with the ZF receiver, which explains the performance degradation of this receiver. From Fig. 3, we can also conclude that the proposed receiver gives a better performance than the receiver of [12] due to the use of TSTC, which exploits spatial transmit diversity at the source and relay nodes. For large values of N, the computational complexities of the LSKP, KRF and ZF receivers are respectively given by $\mathcal{O}(M^3P^{K+1}N)$, $\mathcal{O}(M^2P^{K+1}N)$ and $\mathcal{O}(MP^{K+1}N)$. This shows that the proposed algorithm is a little more complex than the other two techniques. Compared to the performance of the receiver of [18], the proposed receiver gives better performances due to the smaller path-loss experienced by the multi-hop system. We can also see the similar performances provided by the DFT and Hadamard codings, illustrating the advantage in exploiting the orthogonality property regardless of the kind of the used unitary matrix.

Fig. 4 shows the SER versus P_T/N_0 for different system configurations, using the DFT coding. In Fig. 4(a), we evaluate the impact of an increase of the number of relays, $K \in \{1, 2, 3, 4\}$. The multiple time-spreading generated by the TSTC, along with the smaller path-loss of each hop when the number of relays is increased, lead to a performance gain that corroborates the effectiveness of the multi-hop scenario. Fig. 4(b) shows the impact of the number R of transmitted data streams and of the time spreading length P_0 at the source. From this figure, one can conclude that increasing P_0, with R fixed, improves the SER, due to a higher time-diversity at the source, at the cost of a smaller transmission rate. On the other hand, increasing R with P_0 fixed leads to higher SERs, due to a larger number of symbols to be estimated. Note that a same value of R/P_0 implies the same transmission rate, with similar SERs.

Fig. 5 shows the channel NMSE versus P_T/N_0 for the individual channels. We compare the NMSE obtained with the random and DFT codings for the proposed receiver and the KRF receiver [12]. The results illustrate the advantage of the system with DFT coding, compared with random coding and with the system [12], in most of the cases. Moreover, the results show a channel estimation improvement for the hops closest to destination. That comes from the recurrent channel estimation that begins with the last hop and ends with the first hop.

V. Conclusion

In this paper, a closed-form semi-blind receiver has been proposed for a multi-hop MIMO relaying system that uses TSTC at the source and the relays, based on a new tensor model called HONTD. Simulation results have shown that a tensor coding with a unitary unfolding and an increase of the number of relays improve significantly the performance. Perspectives of this work include an extension to non-coherent MIMO [19] and OFDM multi-relay systems.

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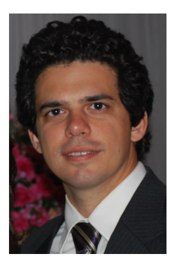

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