# Unsaturated Throughput Analysis of IEEE 802.11 DCF Under $\eta - \mu$ Fading Channel

Heber Rabelo da Silva, Rafael Augusto Pedriali, and Elvio João Leonardo

Abstract—This letter investigates the throughput performance of the IEEE 802.11 DCF protocol in unsaturated traffic conditions, taking into account the signal capture with incoherent addition of interfering signals, and channel fading following the  $\eta - \mu$  model. A set of numerical results on the performance of the protocol is presented. The results show the adjustment flexibility of the proposed channel model and establishes the parameter  $\mu$ as the one responsible for the coarse adjustment of the channel, while the parameter  $\eta$  defines the fine adjustment.

*Index Terms*—Wireless communication, IEEE 802.11 DCF, Throughput, Capture Effect,  $\eta - \mu$  fading.

## I. INTRODUCTION

THE most successful existing commercial implementation of a Wireless Local Area Networks (WLAN) is specified by the IEEE 802.11 standards, which in recent years has grown considerably, making it essential for the connectivity of mobile users. The IEEE 802.11 protocol defines the procedures and services required from the stations, ensuring that all stations have access to the shared medium through the Medium Access Control (MAC) and Physical Layer (PHY) functions [1]. The standard establishes as access mechanisms the Distributed Coordination Function (DCF) and, optionally, the Point Coordination Function (PCF). The PCF assumes an architecture with centralized control, while the DCF operates in an ad hoc mode. This work is restricted only to the mandatory mechanism, i.e., the DCF. The main objective of the DFC, which is based on the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) scheme, is to perform the data transfer between stations with reduced number of collision or even avoiding them altogether, when the stations are contending for access to the shared medium.

The modeling of the IEEE 802.11 DCF has been extensively studied since the appearance of protocol. The analytical model proposed by Bianchi [2] is a precursor; it uses a Markov modeling of the DCF mode and objectively it can be used to evaluate the saturated throughput for both packet transmission techniques, i.e., 2-way handshake, also known as Basic Access, and 4-way handshake, also known as Request-to-Send/Clear-to-Send (RTS/CTS) access mechanism. For this modeling, it is assumed also a finite number of stations, ideal

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channel, i.e., error-free channel, and constant and independent collision probabilities for each station.

Several works extended the model presented by Bianchi, the most significant are in [3]-[10]. In [3] the authors propose the DCF+, a scheme compatible to DCF, to evaluate the performance of the Transmission Control Protocol (TCP) in WLANs by using an implicit channel reservation mechanism. This reservation is performed with a modification in the Acknowledgement (ACK) duration field, thus turning it into an implicit RTS. This new mechanism is analyzed as an extension to Bianchi's model, assuming the same hypothesis as to consider a limit on the number of retransmissions and to establishing a maximum size for the contention window (CW). In [4], [5] the authors assume that while the channel remains busy, the backoff counter remains frozen. However, in [4] it is considered that a particular station can access the medium without activating the backoff after the station has performed a successful transmission. In [6], [7] the authors investigate the saturation throughput in a propagation channel prone to errors due to transmission failures. However in [7] the authors consider the saturated traffic based on the IEEE 802.11a protocol, admitting ACK frames loss due to channel errors, whereas in [6] the lost of ACK frames is disregarded. In [8] the authors evaluate the performance of IEEE 802.11 DCF protocol considering unsaturated traffic and ideal transmission channel, thus expanding the Bianchi modeling with the addition of an idle state in the two-dimensional Markov chain. In [9] the authors extend the work presented in [8] including non-ideal transmission, capture effect of the received signals and a Rayleigh fading channel. In [10] the authors extend the analytical model presented in [9] considering the fading channel modeled by the Hoyt, Rice and Nakagami-m distributions.

In real conditions, the propagated signal degrades during its path. This fading has random characteristics, and therefore the received signal needs to be modeled using a statistical approach; this statistical model is represented by the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF).

In this letter, the channel throughput of the IEEE 802.11 DCF is evaluated. This work extends the results previously mentioned and it is performed assuming a scenario prone to errors, unsaturated traffic conditions and signal capture effect with incoherent addition of the interfering signals under  $\eta - \mu$  fading model [11].

The rest of the letter is organized as follows. Section II presents the analytical model for the IEEE 802.11 DCF. Section III discusses the throughput performance of the IEEE

802.11 protocol, while Section IV presents the  $\eta - \mu$  fading model. The numerical results and conclusions are presented in Sections V and VI, respectively.

# II. ANALYTICAL MODEL

The IEEE 802.11 provides two methods of access to the medium: a distributed method, mandatory, named DCF; and a centralized method, known as PCF, noting that these two methods may coexist. DCF uses a 2-way handshake technique to packet transmition, also known as Basic Access and a 4-way handshake technique, known as RTS/CTS access mechanism.

In the 2-way handshake mechanism, each station, before starting the transmission of a packet, checks the activity of the channel; if the channel remains idle for a period of time at least equal to the Distributed Inter Frame Space (DIFS), the packet is transmitted immediately; if the channel is sensed busy, the station waits a random amount of time which is called backoff time and it is expressed in terms of an integer number of time slots, and chosen within the range  $[0, CW_i - 1]$ , where  $CW_i$ is the size of the contention window at stage i, i = 0, ..., m; while in backoff stage, the station decrements the backoff time counter only if it senses the channel idle; it then tries to transmit as soon as the counter reaches zero. The receiving station, on successfully receiving the packet sent, sends a short ACK frame to confirm the successful transmission. In case of transmission failure (collision or channel error), the transmitting station increments the backoff stage *i*, doubles the contention window  $(CW_i = 2^i CW_{min}, i = 0, ..., m \text{ and } CW_{min}$ being the initial contention window size when i = 0 and waits until the backoff counter reaches zero again in order to attempt a new transmission. This cycle is completed when the packet is received, or the maximum number of retransmissions is reached.

In the 4-way handshake mechanism, when the medium is sensed idle for a DIFS, an RTS frame is sent by the transmitting station to the receiving station requesting permission to transmit. The receiving station replies with a CTS frame, which means that the channel is reserved and transmission can start. If the CTS is not received by the transmitter, a failure is assumed and a retransmission is scheduled using the exponential backoff algorithm explained earlier. The time between RTS, CTS, data and ACK frames is defined as the Short Interframe Space (SIFS).

The operation of the IEEE 802.11 DCF considered in the present work has been modeled by a Markov chain in [9], and aims to provide equal access of the shared medium for all stations. The model assumes that collisions and channel errors are statistically independent events which occur with probabilities  $P_{col}$  and  $P_e$ , respectively. Therefore, a packet is successfully transmitted if neither collision nor channel errors occurs. Thus, a successful transmission occurs with probability  $P_s = (1 - P_{col})(1 - P_e)$ , while the probability of transmission failure is given by  $P_{eq} = P_e + P_{col} - P_{col}P_e$ .

# III. THROUGHPUT ANALYSIS WITH CAPTURE EFFECT

The propagation of radio signals suffers from fading, which will be addressed later, and from interference from other sources that are using the same channel or frequency. A number of different interfering signals may combine at the receiver antenna, producing either coherent or incoherent addition.

Coherent addition occurs if the carriers have the same frequency and the phase fluctuations are insignificant during a given time interval  $t_w$ . The conditions for coherent addition to occur in real networks are uncommon, and therefore will be disconsidered in this work.

For incoherent addition, the random phase fluctuations are significant due to the mutually independent modulation of each signal. Thus, assuming that a component *i* of the interfering signal at the receiver antenna is described by the phasor  $x_i(t) = \operatorname{Re}\{r_i(t)e^{j[\omega_c t + \phi_i(t)]}\}$  in which  $r_i(t)$ ,  $\phi_i(t)$  are the random envelope and phase, respectively, and  $\omega_c$  is the carrier's angular frequency, the power observed during the evaluation period is a sum of powers of individual signals  $w_i$ , i.e.,  $w_n(t) = \sum_{i=1}^n \overline{x_i(t)x_i^*(t)} = \sum_{i=1}^n w_i(t)$ , in which subscripts *i* and *n* are used to represent the individual and aggregated variables, respectively, and  $x_i^*(.)$  is the complex conjugate of phasor  $x_i(.)$ .

In the wireline medium, a transmission is considered successful if there is no signal overlap at the receiver. However, in wireless networks the receiver can be captured by a test packet even with overlapping transmissions if the power ratio between the desired signal  $w_s$  and the joint interference signal (resulting from *n* interfering station)  $w_n$  is greater than a given threshold  $z_0$  during a given period of time  $t_w$  to lock the receiver [10], with  $0 < t_w < t$ , in which *t* is the packet transmission time.

Since only packets for which the power ratio is above the capture threshold can be received, it is considered that, for the packet not to be captured, i.e., destroyed, the ratio  $w_s/w_n \le z_0$  during  $t_w$  for n > 0. Evidently both  $z_0$  and  $t_w$  are established based on modulation techniques and signal encoding employed by the network, which is outside the scope of the present work.

In wireless channels, propagated signals are usually modelled with random behavior. In such a case the capture effect is explored through the signal-to-interference ratio (SIR) which is expressed by

$$Z \triangleq \frac{W_s}{W_n} \ge 0,\tag{1}$$

in which  $W_s \ge 0$  is the desired signal power, and  $W_n \ge 0$  is the interference signal power.

Considering  $W_s$  and  $W_n$  as statistically independent random variables, the PDF of the ratio Z can be expressed as [12]

$$f_{Z}(z) = \int_{0}^{\infty} y f_{Z_{W_{s}}}(zy) f_{Z_{W_{n}}}(y) dy,$$
 (2)

in which  $f_{Z_{W_s}}(.)$  and  $f_{Z_{W_n}}(.)$  are the PDFs of the desired signal power and the interference power, respectively. The CDF is expressed as [12]

$$F_Z(z_0) = \operatorname{Prob}\left\{\frac{W_s}{W_n} \le z_0\right\} = \int_0^{z_0} f_Z(z) dz.$$
(3)

Analyzing (3) one may notice that the CDF defines the power ratio between the desired and the interference signals falling below the capture threshold  $(z_0)$ , i.e., a situation in which the test packet can not be received. Hence, if the number

of actively interfering stations is set of n, the conditional probability of capture may be expressed by

$$P_{capt}(z_0|n) = 1 - F_Z(z_0).$$
(4)

Assuming an environment with N stations generating at most n-1 interfering packets, the unconditional probability of capture of a packet is defined as [9]

$$P_{capt}(z_0) = \sum_{n=1}^{N-1} {N \choose n+1} \delta^{n+1} (1-\delta)^{N-n-1} P_{capt}(z_0|n), \quad (5)$$

in which  $\delta$  is the probability that a station starts a transmission in a randomly chosen time slot and it is calculated in the following paragraph. Assume that the time between packet arrivals is exponentially distributed and its average value expressed by  $\frac{1}{\lambda}$ , in which the parameter  $\lambda$  represents the offered load related to each station. Therefore, a stochastic Poisson process is used as traffic model and the probability that there is at least one packet waiting transmission in the buffer is given as [9]

$$q = 1 - e^{-\lambda E\{S_{ts}\}},\tag{6}$$

in which  $E{S_{ts}}$  is the expected time per slot and it is calculated later in this section.

From the analysis of events and stationary transition probabilities of the Markov chain model proposed in [9] and adopted in [10], the following set of nonlinear equations can be obtained:

$$\begin{cases} P_t = 1 - (1 - \delta)^N \\ P_s = \frac{N\delta(1-\delta)^{N-1} + P_{capt}(z_0)}{P_t} \\ P_{col} = 1 - (1 - \delta)^{N-1} - P_{capt}(z_0) \\ E\{S_{ts}\} = (1 - P_t)\sigma + P_t(1 - P_s)T_c + P_tP_s(1 - P_e)T_s \\ + P_tP_sP_eT_e \\ \delta = \frac{2(2P_{eq}-1)q}{2(q-1)(P_{eq}-1)(2P_{eq}-1) - q[1-2P_{eq}-CW(P(1+(2P_{eq})^m)-1)]}, \end{cases}$$
(7)

in which  $P_t$  the probability that at least one transmission occurs in a given time interval,  $P_s$  the conditional probability that a successful transmission occurs,  $P_{col}$  is the probability that a packet collision occurs,  $T_c$  is the time spent when a channel error occurs due to collision,  $T_s$  is the time spent when a successful transmission occurs, and  $T_e$  is the time spent when a channel error occurs. Thus, the system throughput is defined by [9]

$$S = \frac{P_t P_s (1 - P_e) E\{PL\}}{(1 - P_t)\sigma + P_t (1 - P_s)T_c + P_t P_s (1 - P_e)T_s + P_t P_s P_e T_e},$$
(8)

in which  $\sigma$  is the time of idle channel,  $E\{PL\}$  defines the average packet payload length, and the average times for the 2-way handshaking transmission mechanism are calculated as [9]

$$\begin{cases} T_c = H + PL + ACK_{timeout}, \\ T_s = H + PL + SIFS + 2\tilde{\tau} + ACK + DIFS, \\ T_e = H + PL + ACK_{timeout}, \end{cases}$$
(9)

in which H is the combined PHY and MAC headers duration,  $\tau$  is the worst case propagation delay and  $\tilde{\tau} = \tau/t$  its normalised version.

# IV. Analytical Results for $\eta - \mu$ Fading Model

The  $\eta - \mu$  fading model contemplates a non-line-of-sight (NLOS) environment, in which the resulting signal is composed by *n* sets of reflected waves propagating in inhomogeneous environment [11]. The resultant signal envelope PDF is expressed by

$$f_R(r) = \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}r^{2\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}\hat{r}^{2\mu+1}}\exp\left(-2\mu h\frac{r^2}{\hat{r}^2}\right)I_{\mu-\frac{1}{2}}\left(2\mu H\frac{r^2}{\hat{r}^2}\right),\tag{10}$$

in which  $\Gamma(.)$  is the Gamma function [13, Eq. 6.1.1],  $I_{\nu}(.)$  is the modified Bessel function of the first kind and order  $\nu$  [13, Eq. 9.6.10], and  $\hat{r} = \sqrt{E\{r^2\}}$  is the root mean square value (rms) of r. Assuming the Format 1 defined in [11],  $\eta > 0$  is the power ratio between the scattered waves inphase and quadrature components of each set of multipath waves and  $\mu = \frac{E^2\{r^2\}}{V(r^2)} \frac{1+\eta^2}{(1+\eta)^2}$ , in which  $E\{.\}$  and  $V\{.\}$  are the expectation and variance operators, respectively. Given that  $H/h = (1 - \eta)/(1 + \eta)$ , then

$$h = \frac{(1+\eta)^2}{4\eta}$$
 and  $H = \frac{1-\eta^2}{4\eta}$ . (11)

The  $\eta - \mu$  model is a general fading distribution and includes the Nakagami-*m*, Rayleigh and Hoyt distributions as special cases. Using the standard approach to calculate the ratio of random variables as described by (2) and replacing (10) in (2), the PDF of the random variable Z is given by

$$f_{Z}(z) = \int_{0}^{\infty} \frac{16\pi \mu_{x}^{\mu_{x}+\frac{1}{2}} \mu_{y}^{\mu_{y}+\frac{1}{2}} h_{x}^{\mu_{x}} h_{y}^{\mu_{y}} y^{2\mu_{x}+2\mu_{y}+1} z^{2\mu_{x}}}{\Gamma(\mu_{x})\Gamma(\mu_{y})\hat{x}^{2\mu_{x}+1}\hat{y}^{2\mu_{y}+1} H_{x}^{\mu_{x}-\frac{1}{2}} H_{y}^{\mu_{y}-\frac{1}{2}}} \\ \times \exp\left[\frac{-2\mu_{x}h_{x}}{\hat{x}^{2}} (zy)^{2}\right] \exp\left(\frac{-2\mu_{y}h_{y}}{\hat{y}^{2}} y^{2}\right)$$
(12)  
$$\times I_{\mu_{x}-\frac{1}{2}} \left[\frac{2\mu_{x}H_{x}}{\hat{x}^{2}} (zy)^{2}\right] I_{\mu_{y}-\frac{1}{2}} \left(\frac{2\mu_{y}H_{y}}{\hat{y}^{2}} y^{2}\right) dy.$$

Using [14, Eq. 8.445] and [16] to expand in series the second Bessel function in (12), changing the integration variable to  $t = y^2$ , and changing the integration order, then the PDF can be expressed as

$$f_{Z}(z) = \sum_{i=0}^{\infty} \int_{0}^{\infty} \frac{8\pi \mu_{x}^{\mu_{x}+\frac{1}{2}} \mu_{y}^{2\mu_{y}+2i} h_{x}^{\mu_{x}} h_{y}^{\mu_{y}} H_{y}^{2i} t^{\mu_{x}+2\mu_{y}+2i-\frac{1}{2}} z^{2\mu_{x}}}{i! \Gamma(\mu_{x}) \Gamma(\mu_{y}) \Gamma(\mu_{y}+\frac{1}{2}+i) \hat{x}^{2\mu_{x}+1} \hat{y}^{4\mu_{y}+4i} H_{x}^{\mu_{x}-\frac{1}{2}}}{\hat{x}^{2}} t \left[ \exp\left(\frac{-2\mu_{y} h_{y}}{\hat{y}^{2}} t\right) I_{\mu_{x}-\frac{1}{2}} \left[ \frac{2\mu_{x} H_{x} z^{2}}{\hat{x}^{2}} t \right] dt.$$
(13)

Solving the integral and after some algebraic manipulations, using [14, Eqs. 8.331.1, 8.335.1, 8.384.1], [15, Eqs. 2.15.3.2] and [16] thus the final expression is obtained

$$f_{Z}(z) = \frac{2u^{2\mu_{x}}z^{4\mu_{x}-1}}{h_{x}^{\mu_{x}}h_{y}^{\mu_{y}}(1+uz^{2})^{2\mu_{x}+2\mu_{y}}}$$

$$\times \sum_{i=0}^{\infty} \frac{1}{(i+\mu_{y})B(\mu_{y},i+1)B(2i+2\mu_{y},2\mu_{x})} \left[\frac{H_{y}}{h_{y}(1+uz^{2})}\right]$$

$$\times_{2} F_{1}\left[i+\mu_{x}+\mu_{y},i+\mu_{x}+\mu_{y}+\frac{1}{2},\mu_{x}+\frac{1}{2},\left(\frac{H_{x}}{h_{x}}\frac{uz^{2}}{1+uz^{2}}\right)^{2}\right].$$
(14)

in which B(.) is the Beta function [13, Eq. 6.2.2],  $_2F_1(.)$  is the Gauss hypergeometric function [13, Eq. 15.1.1] and

$$u = \frac{\mu_x h_x \hat{y}^2}{\mu_y h_y \hat{x}^2}.$$
 (15)

Assuming incoherent addition of the interfering signals, where the interfering signal is composed of n independent and identically distributed (i.i.d.) signals, the CDF of the random variable Z, defined in (3), can be expressed by

$$F_{Z}(z) = \int_{0}^{z} \int_{0}^{\infty} \frac{16\pi \mu_{x}^{\mu_{x}+\frac{1}{2}} \mu_{y}^{\mu_{y}+\frac{1}{2}} h_{x}^{\mu_{x}} h_{y}^{\mu_{y}} y^{2\mu_{x}+2\mu_{y}+1} t^{2\mu_{x}}}{\Gamma(\mu_{x})\Gamma(\mu_{y})\hat{x}^{2\mu_{x}+1}\hat{y}^{2\mu_{y}+1} H_{x}^{\mu_{x}-\frac{1}{2}} H_{y}^{\mu_{y}-\frac{1}{2}}} \\ \times \exp\left[\frac{-2\mu_{x}h_{x}}{\hat{x}^{2}}(ty)^{2}\right] \times \exp\left(\frac{-2\mu_{y}h_{y}}{\hat{y}^{2}}y^{2}\right)$$
(16)  
$$\times I_{\mu_{x}-\frac{1}{2}} \left[\frac{2\mu_{x}H_{x}}{\hat{x}^{2}}(ty)^{2}\right] I_{\mu_{y}-\frac{1}{2}} \left(\frac{2\mu_{y}H_{y}}{\hat{y}^{2}}y^{2}\right) dy dt.$$

Using [14, Eq. 8.445] to expand in series the first Bessel function in (16), changing the integration variable to  $w = t^2$ , and changing the integration order, then the CDF can be expressed as

$$F_{Z}(z) = \sum_{i=0}^{\infty} \int_{0}^{\infty} \int_{0}^{z^{2}} \frac{8\pi \mu_{x}^{2\mu_{x}+2i} \mu_{y}^{\mu_{y}+\frac{1}{2}} h_{x}^{\mu_{x}} h_{y}^{\mu_{y}} H_{x}^{2i} w^{2\mu_{x}+2i-1} y^{4\mu_{x}+2\mu_{y}+4i}}{i! \Gamma(\mu_{x}) \Gamma(\mu_{y}) \Gamma(\mu_{x}+\frac{1}{2}+i) H_{y}^{\mu_{y}-\frac{1}{2}} \hat{x}^{4\mu_{x}+4i} \hat{y}^{2\mu_{y}+1}} \\ \times \exp\left[-2\left(\frac{\mu_{x} h_{x}}{\hat{x}^{2}} w + \frac{\mu_{y} h_{y}}{\hat{y}^{2}}\right) y^{2}\right] I_{\mu_{y}-\frac{1}{2}} \left(\frac{2\mu_{y} H_{y}}{\hat{y}^{2}} y^{2}\right) dw dy,$$
(17)

by using [13, Eqs. 6.1.22, 6.5.3, 6.5.4, 6.5.29], [14, Eqs. 8.335.1, 8.356.3], [15, Eqs. 1.3.2.3, 2.15.3.2], [16] and some algebraic manipulations for solving the double-integral in (17), a final expression is obtained as

$$\begin{split} F_{Z}(z) &= \\ \frac{(uz)^{2\mu_{x}}}{h_{y}^{\mu_{y}}(1+uz)^{2\mu_{x}+2\mu_{y}}} \sum_{i=0}^{\infty} \frac{(uz)^{i}}{(i+2\mu_{x})B(i+2\mu_{x},2\mu_{y})(1+uz)^{i}} \\ &\times {}_{2}F_{1}\left[\frac{i}{2}+\mu_{x}+\mu_{y},\frac{i}{2}+\mu_{x}+\mu_{y}+\frac{1}{2},\mu_{y}+\frac{1}{2},\left(\frac{H_{y}}{h_{y}}\frac{1}{1+uz}\right)^{2}\right] \\ &\times \left\{1-\frac{1}{i_{c}B(i_{c},\mu_{x})h_{x}^{\mu_{x}}}\left(\frac{H_{x}}{h_{x}}\right)^{2i_{c}} {}_{2}F_{1}\left[1,i_{c}+\mu_{x},i_{c}+1,\left(\frac{H_{x}}{h_{x}}\right)^{2}\right]\right\}, \end{split}$$
(18)

in which  $i_c = \lceil \frac{i+1}{2} \rceil$ ,  $\lceil . \rceil$  is the Ceiling function.

For a interference-limited scenario over  $\eta - \mu$  fading channel, with a restriction on the value of the  $\mu$  parameters for the interference signals, i.e., whenever that the  $\mu_y$  parameter assume a positive integer number, a closed-form expression for the CDF is obtained in [17].

In [18] the authors extend the results previously reported in [17] and considers scenarios with background noise. For this, admits a limited scenario, wherein for either of the desired signals or components of the interfering signals, the  $\mu$  parameters assumes the integer values. Therefore, the CDF derived in [18] is valid for the constraints of the  $\mu$  parameters, and expressed in terms of elementary functions. Because of restriction of the parameters  $\mu$  in [17] and [18], the derivations does not include the Hoyt model, therefore does not directly applies the formulations this work. Although in practice these limitations are not always available, with minor restrictions in arbitrary values of the  $\mu$  parameters, this expression can be used as limits for real estimates.

 TABLE I

 Typical Parameters - IEEE 802.11B DSSS MAC and PHY Layer

packet payload	8160 bits	channel bit rate	1 Mbp/s
MAC header	272 bits	propagation delay $(\tilde{\tau})$	$1 \ \mu s$
PHY header	192 bits	time slot $(\sigma)$	20 µs
ACK frame	304 bits	SIFS	10 µs
RTS frame	160 bits	DIFS	50 µs
CTS frame	112 bits	ACK timeout	300 µs
т	5	$CW_{min}$	8

### V. NUMERIC RESULTS

Using results from this and from previous sections, it is now possible to evaluate the throughput of the IEEE 802.11 DCF under  $\eta - \mu$  fading environment, assuming 2-way handshake transmission mechanism, unsaturated traffic and capture effect with incoherent addition of the interfering signals. The equations described in (5), (7), (8) and (18), added to the network parameters listed in Table I, led to the results expressed in Figs. 1, 2, 3, 4 and 5. Although the parameters listed in Table I are specified for the IEEE 802.11b protocol, other values may be used for the assessment of any protocol with similar MAC layer.

The Figs. indicate the behavior of the throughput *S* as a function of the packet generation rate  $\lambda$ . Basically, the behavior depicted in the Figs. divides the throughput into two regions: a growing region, represented by *S* as a linear function of  $\lambda$ , and a saturation region, with *S* almost constant. The transition between these two regions often occurs with a peak, which becomes more evident as the number of stations increases.

For the  $\eta - \mu$  fading model considered in the Figs., the results for various values of  $\eta = \eta_s = \eta_n$ ,  $\mu = \mu_s = \mu_n$ ,  $\tilde{\tau}$  and  $P_e$  are presented. Also, consider the normalized capture threshold  $\tilde{z_0} \triangleq z_0/(\overline{w_s}/\overline{w_n})$  in which  $\overline{w_s}$  and  $\overline{w_n}$  are average values of  $W_s$  and  $W_n$ , respectively, and the ratio  $\overline{w_s}/\overline{w_n}$  is commonly denoted as average SIR.

For the purpose of illustration, Fig. 1 presents a scenario with N = 4 stations, with  $\eta$  parameters fixed in 0.5 and  $\mu$  variable. Analyzing the curves, it can be seen that curves 1, 2 and 6 have the same parameters, except that curve 1 has no channel error (it is set to 10% in curves 2 and 6), and the capture threshold is set to 24 dB in curve 6 (it is set to 6 dB in curve 1 and 2). Curves 2 to 5 present the variation in  $\mu$  between 0.1 and 3.5 and show that the throughput is directly proportional to the increase of  $\mu$ . Curves 2 and 6 show the influence of  $\tilde{z}_0$  on the channel throughput; when the value of  $\tilde{z}_0$  grows from 6 to 24 dB, the channel throughput decreases.

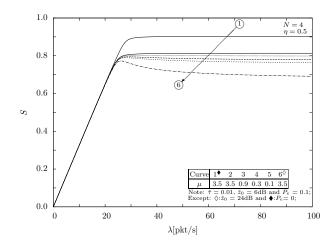


Fig. 1. Throughput for the IEEE 802.11 DCF with 2-way handshake mechanism, incoherent addition of interference signals  $\eta - \mu$  channel, with four stations contending for the medium.

In Fig. 2 a scenario with N = 4 stations is considered, in which the  $\mu$  parameter is fixed at 0.5 and  $\eta$  is variable. The highlighted area presents the behaviour on the channel throughput for the curves 1 to 4. Examining the highlighted area allow us to verify slight variations in throughput for the  $\eta$ parameter varying between  $\eta \to 1$  and  $\eta \to 0$ , which leads to the conclusion that the parameter  $\eta$  defines the fine adjustment of the channel.

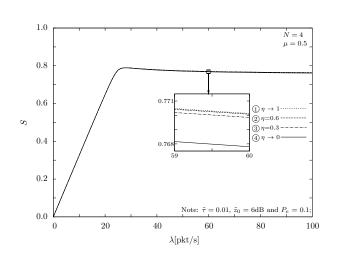
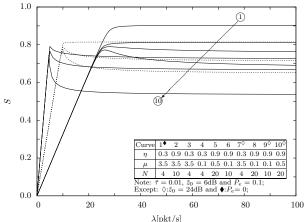


Fig. 2. Throughput for the IEEE 802.11 DCF with 2-way handshake mechanism, incoherent addition of interference signals  $\eta - \mu$  channel, with four stations contending for the medium.

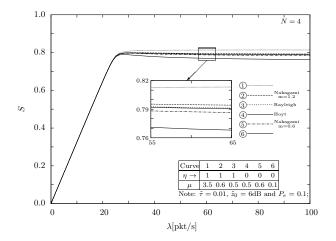
In Fig. 3 scenarios with N = 4, N = 10 and N = 20stations are considered. It is possible to see the influence experienced by the channel throughput when changing the number of stations and with varying  $\eta$  and  $\mu$  parameters. Curves 4, 6 and 9 use the same parameters for N = 4, N = 10and N = 20, respectively. Thus, it can be noticed that the channel throughput is inversely proportional to the number of stations; and the linear growing region presents a stronger growth for higher values of the number of stations N.



 $\lambda [\text{pkt/s}]$ 

Fig. 3. Throughput for the IEEE 802.11 DCF with 2-way handshake mechanism, incoherent addition of interference signals  $\eta - \mu$  channel, with three different numbers of stations contending for the medium.

In Fig. 4 a scenario with N = 4 stations is considered, in which the  $\mu$  parameter is set to 3.5 for curves 1 and 6, while the  $\eta$  parameter varies from  $\eta \to 1$  and  $\eta \to 0$ ; in the other curves the parameters  $\eta$  and  $\mu$  adjusted so that the  $\eta - \mu$  model reduces to traditional fading models such as Hoyt, Rayleigh and Nakagami-m.



Throughput for the IEEE 802.11 DCF with 2-way handshake Fig. 4. mechanism, incoherent addition of interference signals  $\eta - \mu$  channel, with four stations contending for medium. The highlighted area show traditional models such as Hoyt, Nakagami-m and Rayleigh.

In Fig. 5 presents a scenario with N = 20 stations, with  $\eta$ and  $\mu$  parameters fixed at 0.9 and 0.5, respectively. Curves 1 to 6 demonstrate the channel throughput under influence of several capture thresholds  $\tilde{z_0}$ . The highlighted region shows that for the range of  $\tilde{z_0}$  between 6dB and 24dB the channel throughput is inversely proportional to the capture threshold. Curves 1 to 4 show even greater fluctuations in the channel throughput, while the curves 5 and 6 the influence of the  $\tilde{z_0}$ in the channel throughput becomes less expressive.

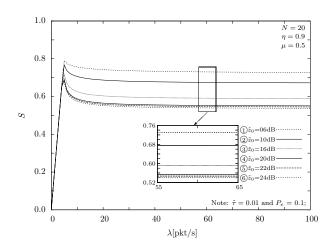


Fig. 5. Throughput for the IEEE 802.11 DCF with 2-way handshake mechanism, incoherent addition of interference signals  $\eta - \mu$  channel, with four stations contending for the medium and capture thresholds varying between  $\tilde{z_0} = 6$ dB and  $\tilde{z_0} = 24$ dB

## VI. CONCLUSION

In this letter, the throughput of the IEEE 802.11 DCF protocol is investigated.

The analytical model presented in [9] was extended by using the generalized  $\eta - \mu$  fading model. Because of the flexibility of the  $\eta - \mu$  distribution, adjusting the parameters  $\eta$  and  $\mu$  allows that a number of different propagation scenarios be considered, with the potential of representing more realistically the real propagation channel. The results show that the parameter  $\mu$ is responsible for the coarse adjustment of the channel, while the parameter  $\eta$  defines the fine adjustment.

The Markov modeling used in this work, as well as the analytical model and its formulations have been fully validated by simulation in [9]. The statistical model used in the fading of signals does not influence the proposed traffic model. The  $\eta - \mu$  fading model adopted in the capture effect extends the work [9], [10] and is fully validated by numerical integration techniques.

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