The Asymmetrical $\eta - \kappa$ Distribution

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***Abstract*— This paper specializes and parameterizes the general result presented elsewhere in the literature in order to propose, fully characterize, and investigate the Asymmetrical $\eta-\kappa$ Distribution. It yields estimators for the involved parameters and uses field measurements to validate the distribution. The Asymmetrical $\eta - \kappa$ Distribution includes, as special cases, important distributions such as Rayleigh, Rice, Hoyt, Nakagami-q, and One-Sided Gaussian. The fact that the Asymmetrical $\eta - \kappa$ Distribution has one more parameter than the well-known distributions renders it more flexible. Of course, in situations in which those distributions included in it give good results a better fit is given by the Asymmetrical $\eta - \kappa$ Distribution. In addition, in many other situations in which these distributions give poor results a good fit may be found through the Asymmetrical $\eta - \kappa$ Distribution. More specifically, its non-monomodal feature finds applications in several circumstances, examples of which are given in this paper.**

***Index Terms*— Fading distributions, Rayleigh distribution, Rice distribution, One-sided Gaussian distribution, Hoyt distribution.**

I. Introduction

THE propagation of energy in a mobile radio environment is characterized by incident waves interacting with surface irregularities via diffraction, scattering, reflection, and absorption. The interaction of the wave with the physical structures generates a continuous distribution of partial waves, with these waves showing amplitudes and phases varying according to the physical properties of the surface. A great number of distributions exist that well describe the statistics of the mobile radio signal. Among those describing the short term signal variation, Rayleigh, Rice, Hoyt, Nakagami-q, and Nakagami-m are the well-known distributions. It has been found that the different distributions yield different fits in different situations. Finding general fading distributions is indeed an old problem that still attracts the attention of the communications researchers [1]–[6]. In [3], a general result is presented in which the in-phase and quadrature components of the fading envelope are *dependent* Gaussian variables with *different non-zero means* and *unequal variances*. The classical Rayleigh, Rice, Hoyt, Nakagami-q, and One-Sided Gaussian density functions are special cases of this general result.

This paper specializes and parameterizes the general result presented in [3] in order to propose, fully characterize, and investigate the Asymmetrical $\eta - \kappa$ Distribution. It yields estimators for the involved parameters and uses field measurements to validate the distribution. The Asymmetrical $\eta-\kappa$ Distribution includes, as special cases, important distributions such as Rayleigh, Rice, Hoyt, Nakagami-q, and One-Sided Gaussian. The fact that the Asymmetrical $\eta - \kappa$ Distribution has one more parameter than the well-known distributions renders it more flexible. Of course, in situations in which those distributions included in it give good results a better fit is given by the Asymmetrical $\eta - \kappa$ Distribution. In addition, in many other situations in which these distributions give poor results a good fit may be found through the Asymmetrical $\eta - \kappa$ Distribution. More specifically, its non-monomodal feature finds applications in several circumstances, examples of which are given in this paper.

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II. The General Result

In his classical paper [1], Nakagami departs from a very general fading model and carries out a series of simplifications, considered to be “sufficiently good enough for engineering problems [1]”, in order to arrive at the well-known Nakagami-m distribution. In the very general model, i.e. without laying hold of the mentioned simplifications, the signal intensity at any observing point is assumed to be composed of a sum of independent random phasors, subject that the in-phase and quadrature components of the sum are normal, i.e., their terms satisfy the conditions of the Central Limit Theorem. These components, therefore, are *dependent* Gaussian variables with different non-zero means and unequal variances. By choosing the in-phase and quadrature axes parallel to the axes of the equiprobability ellipses (i.e., by performing a convenient change of reference phase), the covariance between the Gaussian terms vanishes. The distribution can then be written in terms of *independent* Gaussian variables with different non-zero means and unequal variances. The derivation of the distribution in its very general form is presented in [3], where it is shown in terms of the means and variances of the Gaussian components.

It seems that very little attention has been given to this distribution, maybe because of its rather intricate form of presentation, or for lack of estimators, or, in general, for lack of full characterization. In a work from which the present paper is extracted [7], this general fading distribution is parameterized in terms of the envelope rms value and three power ratios: 1) in-phase dominant component and in-phase scattered wave; 2) quadrature dominant component and quadrature scattered wave; and 3) in-phase term and quadrature term [7]. By taking some specific, but still wide-ranging conditions, simpler forms of the general case can be achieved. In particular, for the power ratios of 1) and 2) assumed to be identical the Symmetrical $\eta-\kappa$ Distribution is attained [8]. For either one of the ratios of 1) or 2) assumed to be nil, the Asymmetrical $\eta-\kappa$ Distribution is accomplished. The concept of symmetry shall be clarified in the text. This paper explores the Asymmetrical $\eta - \kappa$ Distribution.

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III. The Asymmetrical $\eta-\kappa$ Distribution

The Asymmetrical $\eta-\kappa$ Distribution is a general fading distribution that can be used to represent the small-scale variation of the fading signal. For a fading signal with envelope r, phase θ, and normalized envelope $\rho = r/\hat{r}$, in which $\hat{r}=\sqrt{E\left(r^2\right)}$ is the rms value of r, the Asymmetrical $\eta-\kappa$ joint probability density function $p\left(\rho,\theta\right)$ is written as

$$p\left(\rho,\theta\right) = \frac{\sqrt{h}\,\left(1+\kappa\right)\,\rho}{\pi e^{(h+H)\kappa}} \times e^{2\,(h+H)\,\sqrt{\kappa\,(1+\kappa)}\,\rho\,\cos(\theta)-(1+\kappa)\,\rho^2\,(h+H\cos(2\,\theta))} \quad (1)$$

where $h=\frac{2+\eta+\eta^{-1}}{4}$, $H=\frac{\eta^{-1}-\eta}{4}$, $\kappa\geq 0$ is the ratio between the total power of the dominant components and the total power of the scattered waves, and $\eta\geq 0$ is the ratio between the powers of the in-phase term and quadrature term. The normalized envelope probability density function is obtained as

$$p\left(\rho\right)=\frac{2\sqrt{h}\,\left(1+\kappa\right)}{e^{(h+H)\kappa}}\rho\,e^{-h\,(1+\kappa)\,\rho^2}\;\mathrm{M}\left(u,v,0\right) \quad (2)$$

where the $M\left(.,.,.\right)$ function, as defined here, is given by

$$M\left(u,v,\phi\right)=\frac{1}{2\pi}\int_0^{2\pi}\exp\left(u\cos\left(\theta\right)+v\cos\left(2\left(\theta+\phi\right)\right)\right)d\theta \quad (3)$$

with $u = 2\left(h+H\right)\sqrt{\kappa\left(1+\kappa\right)}\,\rho$ and $v = -\left(H\left(1+\kappa\right)\rho^2\right)$. The phase distribution is obtained here in a closed-form manner as (4). , where $\mathrm{erf}\left(.\right)$ is the Gaussian error function. The $n-th$ moment $E\left[\rho^n\right]$ of ρ can be attained in the usual integral manner or in a series expansion given by (5) , where ${}_2F_1\left(.,.;.;.\right)$ is the Gauss hypergeometric function, $\Gamma\left(.\right)$ is the Gamma function. Of course, $E\left[r^n\right]=\hat{r}^nE\left[\rho^n\right]$. In the same way, the cumulative probability function can be obtained in the usual integral manner or in series expansion given by [7]

$$P(\rho)=\sum_{j=0}^{\infty}\sum_{n=0}^{\infty}\sum_{i=0}^{n}\left(\begin{array}{c}n\\ i\end{array}\right)\frac{(-1)^{-i+2j+n}\,4^nh^{-1+i-2j-n}}{\exp\left(\frac{\kappa h}{h-H}\right)} \times\frac{\left(\Gamma\left[1-i+2j+2n\right]-\Gamma\left[1-i+2j+2n,\rho^2\left(1+\kappa\right)h\right]\right)}{\sqrt{\pi}\Gamma\left[1+j\right]\Gamma\left[1+2n\right]\Gamma\left[1-i+2j+2n\right]} \times\Gamma\left[\frac{1}{2}+j+n\right]\left(h-H\right)^{\frac{1}{2}-n}H^{-i+2j+n}\left(h+H\right)^{\frac{1}{2}+n}\kappa^n \quad (6)$$

where $\Gamma\left[a,z\right] = \int_z^{\infty}t^{a-1}\exp\left(-t\right)dt$ is the incomplete Gamma function. The $M\left(.,.,.,.\right)$ function presents some interesting properties related to the Bessel functions. In particular $M\left(u,0,\phi\right)=I_0\left(u\right)$ and $M\left(0,v,\phi\right)=I_0\left(v\right)$, where $I_0\left(.\right)$ is the modified Bessel function of the first kind and order zero. It can be written in terms of the Bessel functions as

$$M\left(u,v,\phi\right)=\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} (\jmath)^n\left(-1\right)^kI_k(-v\,\cos(2\phi))I_n(v\,\sin(2\phi))I_{2(k+n)}(u) \quad (7)$$

For the particular case of the Asymmetrical $\eta-\kappa$ Distribution, in which $\phi=0$,

$$M\left(u,v,0\right)=I_0(u)I_0(v)+2\sum_{k=1}^{\infty}\left(-1\right)^kI_k(v)I_{2k}(u) \quad (8)$$

Fig. 1, for a fixed η ($\eta=0$) and varying κ, Fig. 2, for a fixed η ($\eta=0.5$) and varying κ, and Fig. 3, for a fixed κ ($\kappa=1$) and varying η, show the various shapes of the Asymmetrical $\eta-\kappa$ probability density function $p\left(\rho\right)$.

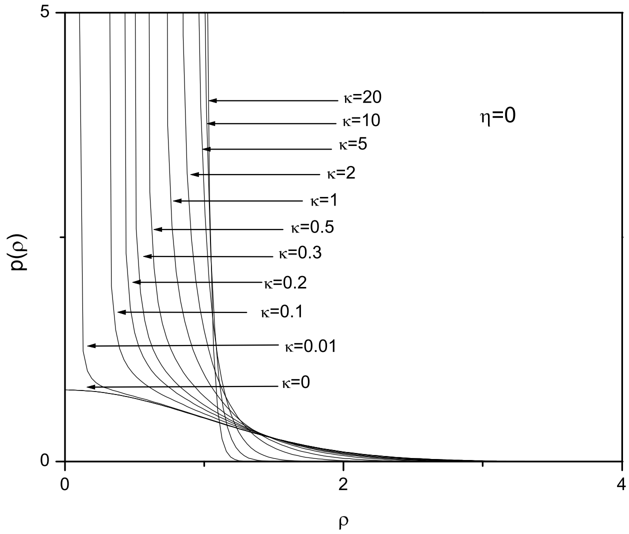

Fig. 1. The Asymmetrical $\eta-\kappa$ probability density function for a fixed η ($\eta=0$).

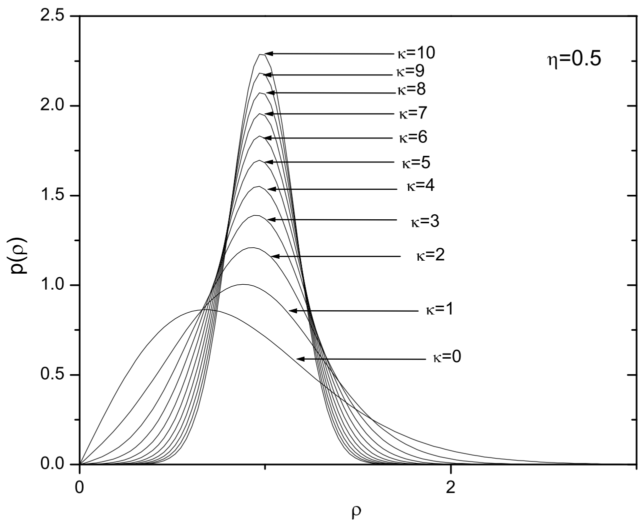

Fig. 2. The Asymmetrical $\eta-\kappa$ probability density function for a fixed η ($\eta=0.5$).

$$p\left(\theta\right)=\frac{e^{-(h+H)\,\kappa}\,\sqrt{h}\,\left(\sqrt{h+H\,\cos(2\,\theta)}+e^{\frac{(h+H)^2\,\kappa\,\cos(\theta)^2}{h+H\,\cos(2\,\theta)}}\,(h+H)\,\sqrt{\pi\kappa}\,\cos(\theta)\,\left(1+\mathrm{erf}(\frac{(h+H)\,\sqrt{\kappa}\,\cos(\theta)}{\sqrt{h+H\,\cos(2\,\theta)}})\right)\right)}{2\,\pi\,(h+H\,\cos(2\,\theta))^{\frac{3}{2}}} \tag{4}$$

$$E\left[\rho^n\right]=\sum_{j=0}^{\infty}\sum_{t=0}^{\infty}\sum_{i=0}^{t}\left(\begin{array}{c}t\\i\end{array}\right)\frac{(-1)^{-i+2\,j+t}\,4^t\,e^{\frac{h\,\kappa}{-h+H}}\,h^{-1+i-2\,j-t-n}\,(h-H)^{\frac{1-2\,t+n}{2}}\,H^{-i+2\,j+t}}{\sqrt{\pi}\,(1+\kappa)^{\frac{n}{2}}\,\Gamma(1+j)\,\Gamma(1+2\,t)\,\Gamma(1-i+2\,j+2\,t)}$$
$$\times(h+H)^{\frac{1}{2}-2\,i+4\,j+3\,t-2\,(-i+2\,j+t)+\frac{n}{2}}\,\kappa^t\,\Gamma(\frac{1}{2}+j+t)\,\Gamma(1-i+2\,j+2\,t+\frac{n}{2}) \tag{5}$$

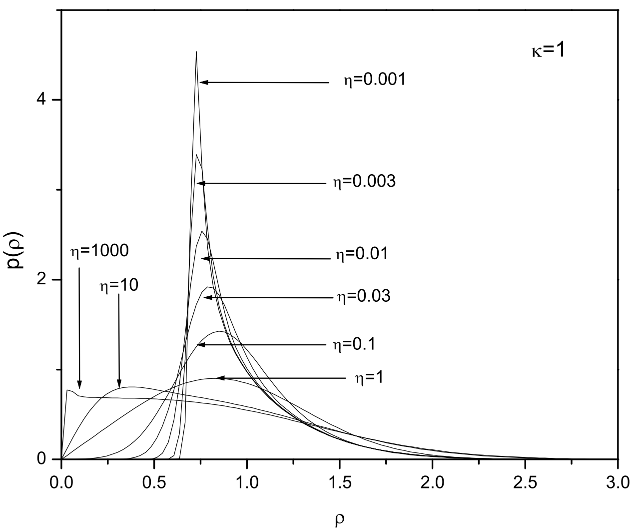

Fig. 3. The Asymmetrical $\eta-\kappa$ probability density function for a fixed κ ($\kappa=1$).

IV. Outline of the Derivation of the Asymmetrical $\eta-\kappa$ Distribution

The fading model for the Asymmetrical $\eta-\kappa$ Distribution considers a signal composed of multipath waves propagating in a non-homogeneous environment. The powers of the in-phase and quadrature scattered waves are assumed to be arbitrary. In the same way, the power of the in-phase component is also assumed to be arbitrary but the power of the quadrature dominant component is assumed to be nil, which explains the reason for it to be called the asymmetrical distribution. Let x and y be independent Gaussian wide sense stationary processes of the in-phase and quadrature components of the propagated wave, respectively. Assume that $\mathrm{E}\left(x\right)=\mu_x$, $\mathrm{E}\left(y\right)=0$, $\mathrm{Var}\left(x\right)=\sigma_x^2$, and $\mathrm{Var}\left(y\right)=\sigma_y^2$, where $E\left(.\right)$ and $Var\left(.\right)$ are the mean and variance operators, respectively. The joint distribution $p\left(x,y\right)$ of x and y is found in the usual manner. The envelope r can be written in terms of the in-phase and quadrature components of the fading signal as $r^2=x^2+y^2$ with $x=r\cos\left(\theta\right)$, $y=r\sin\left(\theta\right)$, and $\theta=\arctan\left(\frac{y}{x}\right)$. Then the joint density $p\left(r,\theta\right)$ of r and θ is obtained by the well-known procedure of transformation of variables. Given the physical model of the distribution, $\eta=\frac{\sigma_x^2}{\sigma_y^2}$ and $\kappa=\frac{\mu_x^2}{\sigma_x^2+\sigma_y^2}$. For the Asymmetrical case then $E\left(r^2\right)=r_{rms}^2=\left(1+\eta^{-1}\right)\left(1+\kappa\right)\sigma_x{}^2$. By carrying out the appropriate substitutions and after long algebraic manipulations the densities as presented in Section III follow.

V. Estimators for the Parameters η and κ

Estimators for the parameters η and κ can be obtained in terms of the moments of the envelope. In particular, $E[\rho^4]$ and $E\left[\rho^6\right]$ can be express*ed in a radical form as functions of η and κ. These equations are then manipulated to produce (9) and (10)

$$\eta=\frac{1-\kappa\pm\sqrt{-2+\mathrm{E}\left[\rho^4\right]-4\,\kappa+2\,\mathrm{E}\left[\rho^4\right]\,\kappa+\mathrm{E}\left[\rho^4\right]\,\kappa^2}}{1+\kappa\mp\sqrt{-2+\mathrm{E}\left[\rho^4\right]-4\,\kappa+2\,\mathrm{E}\left[\rho^4\right]\,\kappa+\mathrm{E}\left[\rho^4\right]\,\kappa^2}} \tag{10}$$

Given $E[\rho^4]$ and $E\left[\rho^6\right]$ (9) yields six possible solutions for κ, but only one will be non-negative and real. Using this value for κ, (10) can be solved to obtain two possible values for η. Therefore, two possible pairs of solutions, (η_1,κ) and (η_2,κ), are attained. Such an ambiguity can be resolved by means of the use of another moment (e.g., the first moment). In such a case: 1) estimate the first moment (E$[\rho]$) with the pair (η_1,κ_1) in (5); 2) estimate the first moment (E$[\rho]$) with the pair (η_2,κ_2) in (5); 3) Estimate the first moment of the measured data; 4) Compare the results of 1) with 3) and 2) with 3); 4) Choose the pair whose corresponding first moment is closest to the one of the measured data. Of course, if the the data follows the Asymmetrical $\eta-\kappa$ distribution in an exact manner, then the smallest difference is nil.

Given a set of measured data for the fading envelope, the practical procedure in order to determinate the distributions parameters η and κ is as follows: 1) Estimate E$[r^2]$, E$[r^4]$, and E$[r^6]$; 2) Using E$[\rho^4]$ = E$[r^4]$/E$[r^2]^2$ and E$[\rho^6]$ = E$[r^6]$/E$[r^2]^3$ in (9) and (10) η and κ are obtained. According to the definition of these parameters as given previously, κ is the ratio between the total power of the dominant components and the total power of the scattered waves, and η is the ratio between the powers of the in-phase term and quadrature term. One important point to raise is that, because the estimators of the Asymmetrical η-κ require the computation of higher order statistics, namely E$[\rho^4]$ and E$[\rho^6]$ [1], a large quantity of data is required for an appropriate convergence. In case a sufficient

[1]For the Nakagami-m parameter the higher order statistics required is E$[\rho^4]$.

$$
\begin{gathered}
144-216\,\mathrm{E}\left[\rho^4\right]+81\,\mathrm{E}\left[\rho^4\right]^2+24\,\mathrm{E}\left[\rho^6\right]-18\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+\mathrm{E}\left[\rho^6\right]^2+\\
\left(864-1296\,\mathrm{E}\left[\rho^4\right]+486\,\mathrm{E}\left[\rho^4\right]^2+144\,\mathrm{E}\left[\rho^6\right]-108\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+6\,\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa+\\
\left(2160-3240\,\mathrm{E}\left[\rho^4\right]+1215\,\mathrm{E}\left[\rho^4\right]^2+360\,\mathrm{E}\left[\rho^6\right]-270\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+15\,\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa^2+\\
\left(2496-4032\,\mathrm{E}\left[\rho^4\right]+1620\,\mathrm{E}\left[\rho^4\right]^2+448\,\mathrm{E}\left[\rho^6\right]-360\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+20\,\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa^3+\\
\left(1296-2520\,\mathrm{E}\left[\rho^4\right]+1215\,\mathrm{E}\left[\rho^4\right]^2+264\,\mathrm{E}\left[\rho^6\right]-270\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+15\,\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa^4+\\
\left(288-720\,\mathrm{E}\left[\rho^4\right]+486\,\mathrm{E}\left[\rho^4\right]^2+48\,\mathrm{E}\left[\rho^6\right]-108\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+6\,\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa^5+\\
\left(16-72\,\mathrm{E}\left[\rho^4\right]+81\,\mathrm{E}\left[\rho^4\right]^2-8\,\mathrm{E}\left[\rho^6\right]-18\,\mathrm{E}\left[\rho^4\right]\,\mathrm{E}\left[\rho^6\right]+\mathrm{E}\left[\rho^6\right]^2\right)\,\kappa^6=0
\end{gathered}
\tag{9}
$$

amount of data is not available, it may be adequate to use one of the alternative fit method as described in Sec. VII.

VI. The Asymmetrical $\eta-\kappa$ Distribution and the Other Fading Distributions

The Asymmetrical $\eta-\kappa$ Distribution is a general fading distribution that includes the Rayleigh, Rice, Hoyt, Nakagami-q, and One-Sided Gaussian distributions as special cases. It may also approximate the Nakagami-m distribution.

A. Rayleigh, Rice, Hoyt, Nakagami-q, and One-Side Gaussian

The Hoyt distribution can be obtained from the Asymmetrical $\eta-\kappa$ Distribution in an exact manner by setting $\kappa=0$ and using the relation $b=\frac{\eta-1}{\eta+1}$, where b is the Hoyt parameter. From the Hoyt distribution the One-Sided Gaussian is obtained for $b\rightarrow\mp1$ ($\eta\rightarrow0$ or $\eta\rightarrow\infty$). In the same way, from the Hoyt distribution the Rayleigh distribution is obtained in an exact manner for $b=0$ ($\eta=1$). The Nakagami-q distribution can be obtained from Asymmetrical $\eta-\kappa$ Distribution in an exact manner by setting $\kappa=0$ and using $q=\eta$, where q is the respective parameter. From the Nakagami-q the One-Sided Gaussian can be obtained for $q\rightarrow0$ or $q\rightarrow\infty$ ($\eta\rightarrow0$ or $\eta\rightarrow\infty$). In the same way, from the Nakagami-q the Rayleigh distribution can be obtained by setting $q=1$ ($\eta=1$). The Rice Distribution can be obtained from the Asymmetrical $\eta-\kappa$ Distribution in an exact manner by setting $\eta=1$ and using $k=\kappa$, where k is the Rice parameter. From the Rice distribution the Rayleigh can be obtained for $k=0$ ($\kappa=0$).

B. Nakagami-m

The Nakagami parameter m can be written in terms of η and κ by recognizing that m is the inverse of the normalized variance of the squared envelope, i.e. $m=Var^{-1}\left(\rho^2\right)$. Using such a definition for the Asymmetrical $\eta-\kappa$ Distribution, it can be shown that

$$
m=\frac{\left(1+\eta^{-1}\right)^2(1+\kappa)^2}{2\left(1+\eta^{-2}+2\left(1+\eta^{-1}\right)\kappa\right)}
\tag{11}
$$

From (11) it can be seen that, apart from the case $m=0.5$, for which $(\eta,\kappa)=(0,0)$ is the only possible solution, an infinite number of Asymmetrical $\eta-\kappa$ curves can be found for the same m parameter. An appropriate choice of $(\eta,\,\kappa)$ pairs may be found that leads to the best Nagakami-m approximation. Interestingly, it can be observed that the minimum of (11) is obtained for $(\eta,\kappa)=(0,0)$, for which $m=0.5$. In this case, the Asymmetrical $\eta-\kappa$ Distribution specializes into the One-Sided Gaussian, as does Nakagami-m.

VII. Application of the $\eta-\kappa$ Distribution

The application of the Asymmetrical $\eta-\kappa$ Distribution implies the estimation of its parameters η and κ (see Section V). On the other hand, it may be possible to use the Asymmetrical $\eta-\kappa$ Distribution by estimating the parameter m and choosing the appropriate $(\eta,\,\kappa)$ pair satisfying (11) that leads to the best fit. In particular, for a given m and κ the remaining parameter η can be chosen as

$$
\begin{aligned}
\eta=\frac{1}{-1+2\,m-2\,\kappa+4\,m\,\kappa-\kappa^2}&\left(1+2\,\kappa-2\,m\,\kappa+\kappa^2\right.\\
&\left.\pm2\,\sqrt{m\left(1-m+2\,\kappa-2\,m\,\kappa+\kappa^2+m\,\kappa^2\right)}\right)
\end{aligned}
\tag{12}
$$

with the usual physical constraint $\eta\geq0$ and *real*. Fig. 4 depicts a sample of the various shapes of the Asymmetrical $\eta-\kappa$ probability distribution function $P(\rho)$ as a function of the normalized envelope ρ for the same Nakagami parameter $m=1.25$. It can be seen that, although the normalized variance (parameter m) is kept constant, the curves are substantially different from each other. Note that the lower tail of the distribution may yield differences in the probability of some orders of magnitude. Note also that the Asymmetrical $\eta-\kappa$ curves can be above or below the Nakagami curve.

VIII. Validation Through Field Measurements

A series of field trials was conducted at the University of Campinas (Unicamp), Brazil, in order to investigate the short term statistics of the fading signal at 1.8GHz [9]. In particular, transmitter and receiver were placed within buildings (*indoor propagation*), and the procedure used is that of the already widely reported in the literature [10].

Through our measurements, it has been observed that the Asymmetrical $\eta-\kappa$ Distribution finds its applications for the cases in which the Rice Distribution is also applicable ($m\geq1$). In the same way, it has found its applicability in those (less frequent) cases in which Hoyt is also applicable ($0\leq$

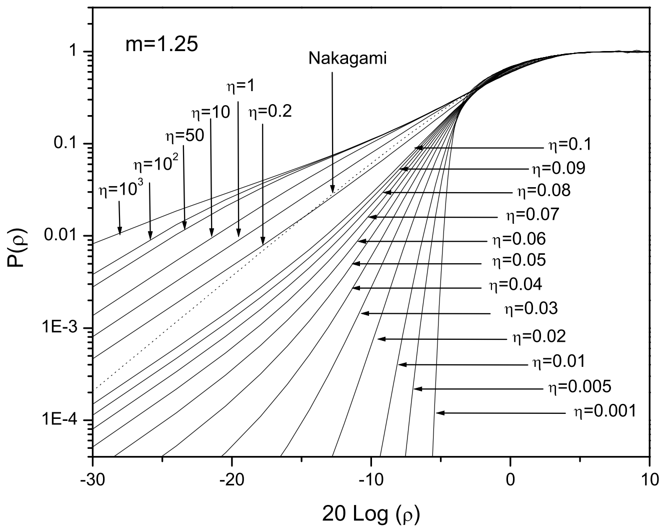

Fig. 4. The Asymmetrical $\eta - \kappa$ probability distribution function for the same Nakagami parameter m (m=1.25).

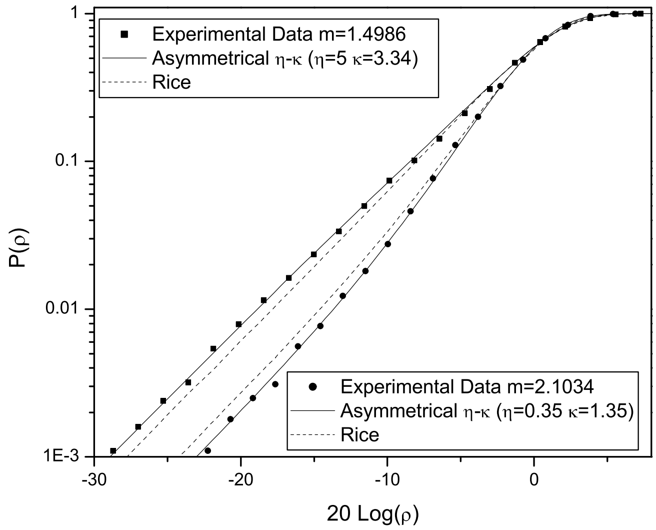

Fig. 5. The Asymmetrical $\eta - \kappa$ distribution function adjusted to data of an indoor propagation measurement at 1.8 GHz conducted at Unicamp.

$m \leq 1$). On the other hand, because of its versatility it yields excellent fit for the cases in which the fit provided by these respective distributions is only moderate. Fig. 5 shows some sample plots illustrating the adjustment by the Asymmetrical $\eta - \kappa$ Distribution as compared to the Rice one. Note, in Fig. 5, that the Rice distribution provides good fit in both cases, although the Asymmetrical $\eta - \kappa$ Distribution gives a better adjustment. Also in Fig. 5 note how the Asymmetrical $\eta - \kappa$ Distribution tends to reproduce or very closely follow the trends (concavity and/or convexity) of the true curve.

A numerical measure of the mean error deviation[2] between the true curve and the distributions chosen to fit the experimental data (namely, Rice and Asymmetrical η-κ) has been calculated for all of the cases. Tab. I shows these for the curves in Fig. 5. In both cases, the error for the Asymmetrical η-κ distribution is lower than that for the Rice distribution, although, the Rice distribution also gives good results. On the other hand, as opposed to the Rice one, the Asymmetrical $\eta - \kappa$ Distribution tends to reproduce or very closely follow the trends (concavity and/or convexity) of the true curve.

[2]The mean error deviation between the measured data x_i and the theoretical value y_i (Rice or Asymmetrical η-κ) is defined as $\epsilon = \frac{1}{N}\sum_{i=1}^{N}\frac{|y_i - x_i|}{x_i}$, where N is the number of points. Other metrics have been tested, e.g. rms deviation, and in all of them the Asymmetrical η-κ presented a better performance.

	Asymmetrical $\eta - \kappa$	Rice
Fig.5 (upper curve)	0.081	0.160
Fig. 5 (lower curve)	0.145	0.383

TABLE I

MEAN ERROR DEVIATION.

IX. CONCLUSIONS

This paper has presented the Asymmetrical $\eta - \kappa$ Distribution, a general fading distribution that includes Rayleigh, Rice, Hoyt, Nakagami-q, and One-Sided Gaussian as special cases. Simulation and field trials have been conducted in order to better characterize this distribution. It has been found that Asymmetrical $\eta - \kappa$ Distribution finds its applications for the cases in which Hoyt as well as Rice distributions are also applicable. Because of its versatility it yields excellent fit for the cases in which the fit provided by these respective distributions is only moderate. It has been observed that the Asymmetrical $\eta - \kappa$ Distribution tends to closely follow the shapes of the true distribution which, sometimes, does not comply with the monomodal behavior of the well-known distributions.

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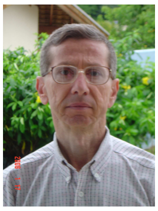

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