

# A DISTANCE MEASURE FOR PERFORMANCE EVALUATION OF SHAPE ADAPTIVE TRANSFORMS

Emilio Carlos Acocella and Abraham Alcaim

**Resumo** - Em recente artigo, foi proposta uma formulação matemática para transformadas 2D adaptativas à forma. Com base nessa formulação, propõe-se neste artigo uma métrica que permite avaliar, teoricamente, o desempenho de uma transformada adaptativa à forma genérica, em relação à uma transformada ótima. Essa métrica é baseada na minimização da diferença média quadrática entre os coeficientes de uma transformada adaptativa à forma e os da transformada ótima. Os resultados experimentais apresentados permitem concluir que a métrica proposta consiste em eficiente instrumento de avaliação de desempenho de transformadas adaptativas à forma e pode ser de particular interesse para aplicações de codificação de vídeo baseada em objeto.

**Palavras-chave:** Codificação baseada em objeto, transformadas adaptativas à forma, codificação de vídeo.

**Abstract** - In a recent work, a mathematical formulation for shape adaptive (SA) 2D transforms was developed. Based on this formulation, we propose in this paper a distance measure that theoretically assesses the performance of a generic SA transform. This measure is based on the minimization of the mean square difference between the coefficients of a given SA transform and those of the optimum SA transform. Experimental results show that the proposed measure is an efficient tool for performance evaluation purposes and may be of major interest for object-based video coding applications.

**Keywords:** Object-based coding, shape adaptive transforms, video coding.

## 1. INTRODUCTION

In object-based video compression, an image sequence is encoded in such a way that objects can be separately decoded and easily manipulated at the receiving end, allowing increased interactivity and compression efficiency. Shape adaptive 2D-transforms represent important tools to this area. In a recent work reported in the literature [1], a mathematical formulation was developed for these transforms. It was shown that with a simple mapping from  $R^2$  to  $R$ , we can represent any type of SA transform by means of a concatenation of three linear operators, which characterize the 1D vertical transforms, the alignment of vertical coefficients and the 1D horizontal transforms. The DCT used in the SA-DCT described in [2] is an example of

the 1D vertical and horizontal transforms. Two different strategies of vertical coefficients alignment are described in [2] and [3].

It is certainly of interest to have useful distance measures to assess the performance of a given SA transform with respect to an optimum transform, such as the Karhunen-Loève transform (KLT). The literature presents solutions to the case of regular (rectangular) transforms. An example is the development reported in [4, p. 176] to the family of sinusoidal transforms. However, such solutions are not applied to the support region of the image segments used in object-based video coding. In this case, the algebraic formulation described in this paper allows the derivation of an appropriate distance measure. Another approach usually employed in the literature is the transform coding gain  $G_{TC}$  over PCM [5]-[7] for each transform individually. The difference to our proposal is that we derive an analytical expression for a distance measure  $\mu$  between a given SA transform and the optimum transform in terms of a mathematical formulation which is specific for shape adaptive transforms [1]. The proposed measure is, therefore, useful for formal manipulations and evaluations of SA transforms based on this mathematical formulation.

Section II of this paper contains the algebraic formulation required for the derivation of the SA-KLT matrix. It is based on the autocorrelation and cross-correlation matrices of the vectors representing the object columns. Then, an expression is obtained for the autocorrelation matrix of a specific class of signals: the wide sense stationary 2D  $1^{\text{st}}$ -order Markov processes, with statistically independent horizontal and vertical coefficients. Section III is devoted to the proposal of a distance measure between SA transforms, which allows their comparison and the selection of the more efficient ones as compared to the SA-KLT. This resource eliminates a gap in the theory of SA transform coding. Because this tool was not available, previous works evaluate the performance of SA transforms either on experimental basis or in terms of the transform gain over PCM. The simulation results presented in Section IV confirm the usefulness of the proposed distance measure. Section V highlights some important aspects, which are specific of the proposed metric, and summarizes the main contributions of the paper.

## 2. THE OPTIMUM SA TRANSFORM

Due to its well known coefficients decorrelation and energy compaction properties [4], the KLT is taken as the optimum transform for comparative purposes. For this reason, it is of interest to derive an expression for the discrete SA-KLT  $\Phi^*$ . Since the KLT is a unitary transform, its inverse is the conjugate transpose, i.e.,

$$\underline{y} = \Phi^{*1} \underline{x} \Leftrightarrow \underline{x} = \Phi \underline{y} \quad (1)$$

where  $\underline{x}$  is an N-dimensional random vector and  $\underline{y}$  is its KLT. The rows of the KLT matrix are the complex conjugate vectors of the normalized eigenvectors of  $R_x$ , the

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autocorrelation matrix of  $x$ . Therefore,  $\phi_k$ , the  $k$ -th column of  $\Phi$ ,  $1 \leq k \leq N$ , is the  $k$ -th normalized eigenvector of  $\mathbf{R}_x$  and, consequently,

$$\mathbf{R}_x = \mathbf{E}\{\underline{y} \underline{y}^H\} = \mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n, \dots, \lambda_N) \quad (2)$$

where  $\lambda_i$ ,  $1 \leq i \leq N$ , are the eigenvalues of  $\mathbf{R}_x$ .

These concepts can be used to obtain the optimum SA transform. Before, however, we will represent the 2-D real intensities of an arbitrarily shaped object as a vector. This representation can be obtained from the mathematical formulation reported in [1]. Firstly, the original object is represented as a rectangular one with dimensions  $M \times M$  by the matrix

$$\mathbf{F} = \begin{bmatrix} \underline{\mathbf{0}}_{i1} & \underline{\mathbf{0}}_{i2} & \dots & \underline{\mathbf{0}}_{iM} \\ \underline{\mathbf{f}}_{01} & \underline{\mathbf{f}}_{02} & \dots & \underline{\mathbf{f}}_{0M} \\ \underline{\mathbf{0}}_{f1} & \underline{\mathbf{0}}_{f2} & \dots & \underline{\mathbf{0}}_{fM} \end{bmatrix} \quad (3)$$

where the vector  $\underline{\mathbf{f}}_{0j}$ ,  $j = 1, \dots, M$ , contains the  $N_j$  pixels of the  $j$ -th object column,  $\underline{\mathbf{0}}_{ij}$  is an  $n_{ij}$ -dimensional null vector representing the initial elements of the  $j$ -th column that do not belong to the object, and  $\underline{\mathbf{0}}_{fj}$  is an  $n_{fj}$ -dimensional null vector representing the final elements of the  $j$ -th column that do not belong to the object. Note that  $n_{ij} + N_j + n_{fj} = M$ .

From  $\mathbf{F}$  we can define a vector  $\underline{\mathbf{f}}_0$  in  $\mathbf{R}^N$ , where  $N = N_1 + \dots + N_M$ , obtained from the non-null vectors of the columns of  $\mathbf{F}$ . It is given by

$$\underline{\mathbf{f}}_0 = \begin{bmatrix} \underline{\mathbf{f}}_{01} \\ \underline{\mathbf{f}}_{02} \\ \vdots \\ \underline{\mathbf{f}}_{0M} \end{bmatrix} \quad (4)$$

The discrete SA-KLT matrix can be obtained from the autocorrelation matrix  $\mathbf{R}_{f_0}$  of this vector, which is defined by

$$\mathbf{R}_{f_0} = \mathbf{E}\{\underline{\mathbf{f}}_0 \underline{\mathbf{f}}_0^H\} \quad (5)$$

Substituting (4) into (5), and defining  $\mathbf{R}_{f_{0i} f_{0j}} = \mathbf{E}\{\underline{\mathbf{f}}_{0i} \underline{\mathbf{f}}_{0j}^H\}$ ,  $i, j = 1, \dots, M$ , as the correlation matrices of the vectors  $\underline{\mathbf{f}}_{0j}$ ,  $j = 1, \dots, M$ , we then get

$$\mathbf{R}_{f_0} = \begin{bmatrix} \mathbf{R}_{f_{01} f_{01}}(s_{11}) & \mathbf{R}_{f_{01} f_{02}}(s_{12}) & \dots & \mathbf{R}_{f_{01} f_{0M}}(s_{1M}) \\ \mathbf{R}_{f_{02} f_{01}}(s_{21}) & \mathbf{R}_{f_{02} f_{02}}(s_{22}) & \dots & \mathbf{R}_{f_{02} f_{0M}}(s_{2M}) \\ \dots & \dots & \dots & \dots \\ \mathbf{R}_{f_{0M} f_{01}}(s_{M1}) & \mathbf{R}_{f_{0M} f_{02}}(s_{M2}) & \dots & \mathbf{R}_{f_{0M} f_{0M}}(s_{MM}) \end{bmatrix} \quad (6)$$

In this expression,  $s_{kj} = n_{ij} - n_{ik}$  is the relative shift between the  $k$ -th and  $j$ -th columns, where  $n_{ik}$  and  $n_{ij}$  are, respectively, the number of pixels that do not belong to the object in the beginning of these columns. Note that the cross-correlation matrix of the random vectors  $\underline{\mathbf{f}}_{0k}$  and  $\underline{\mathbf{f}}_{0j}$  is, in fact, a function of this relative shift and, for this reason, it will be denoted by  $\mathbf{R}_{f_{0k} f_{0j}}(s_{kj})$ .

A 1-D 1<sup>st</sup>-order Markov process with correlation coefficient equal to  $\alpha$  is an autoregressive stationary process  $x(n)$  that can be represented by

$$x(n) = \alpha x(n-1) + v(n) \quad (7)$$

where  $v(n)$  is a white noise. Successively applying this equation  $\tau$  times, we get

$$x(n) = \alpha^\tau x(n-\tau) + \sum_{k=0}^{\tau-1} \alpha^k v(n-k) \quad (8)$$

Since  $x(n)$  is a stationary process, the correlation between  $x(n)$  and  $x(n-\tau)$  does not depend on  $n$ . It can be expressed by Note that  $R(0) = \mathbf{E}\{x(n)x(n)\}$  and that the second term in (9)

$$\begin{aligned} R(\tau) &= \mathbf{E}\{x(n)x(n-\tau)\} = \\ &= \mathbf{E}\left\{\left[\alpha^\tau x(n-\tau) + \sum_{k=0}^{\tau-1} \alpha^k v(n-k)\right]x(n-\tau)\right\} = \\ &= \alpha^\tau \mathbf{E}\{x(n-\tau)x(n-\tau)\} + \sum_{k=0}^{\tau-1} \alpha^k \mathbf{E}\{x(n-\tau)v(n-k)\} = \\ &= \alpha^\tau R(0) \end{aligned} \quad (9)$$

is zero because  $x(n)$  and  $v(n)$  are uncorrelated processes.

Now consider a model which is often used for image analysis: a wide sense stationary and separable ( $x(n_1, n_2) = x(n_1)x(n_2)$ ) 1<sup>st</sup>-order bidimensional Markov process, with horizontal and vertical correlation coefficients (i.e., between columns and between rows) equal to  $\beta$  and  $\alpha$ , respectively. For simplicity, we assume 2D separability and stationarity as in references [6], [9] and [10]. A more realistic 2D non-separable, noncausal, and non-stationary model has been considered already for rectangular-shaped fields [11]-[13], but it has not been done yet for arbitrarily-shaped fields.

Using an algebraic derivation similar to the one described for the 1-D case, we obtain

$$\begin{aligned} R(\tau_1, \tau_2) &= \mathbf{E}\{x(n_1, n_2)x(n_1-\tau_1, n_2-\tau_2)\} = \\ &= \alpha^{\tau_1} \beta^{\tau_2} R(0,0) \end{aligned} \quad (10)$$

where  $R(0,0) = \mathbf{E}\{x(n_1, n_2)x(n_1, n_2)\}$ . Applying this model to the sub-matrices in (6) we get

$$\mathbf{R}_{f_{0i} f_{0j}}(s_{ij}) = \mathbf{R}(0,0) \beta^{|i-j|} \begin{bmatrix} \alpha^{|-s_{ij}|} & \alpha^{|-s_{ij}-1|} & \alpha^{|-s_{ij}-2|} & \dots & \alpha^{|-s_{ij}-(N_i-1)|} \\ \alpha^{|-s_{ij}+1|} & \alpha^{|-s_{ij}|} & \alpha^{|-s_{ij}-1|} & \dots & \alpha^{|-s_{ij}-(N_i-2)|} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha^{|-s_{ij}+(N_i-1)|} & \alpha^{|-s_{ij}+(N_i-2)|} & \alpha^{|-s_{ij}+(N_i-3)|} & \dots & \alpha^{|-s_{ij}+(N_i-N_i)|} \end{bmatrix} \quad (11)$$

Replacing this expression into (6) we obtain the autocorrelation matrix  $\mathbf{R}_{f_0}$  of  $f_0$ . The eigenvectors of this matrix are the rows of the SA-KLT matrix.

### 3. A DISTANCE MEASURE FOR EVALUATION OF SA TRANSFORMS

The distance measure proposed in this paper to evaluate a given unitary transform  $\mathbf{T}_i$  is given by the mean square difference between the transform coefficients of the original object represented by  $\underline{\mathbf{f}}_0$  and those of the SA-KLT  $\Phi^{*t}$  applied to the same vector  $\underline{\mathbf{f}}_0$ . It is defined by

$$\mathbf{E}\{d^2(\mathbf{T}_i \underline{\mathbf{f}}_0 - \Phi^{*t} \underline{\mathbf{f}}_0)\} \quad (12)$$

where the power of the coefficients of  $\mathbf{T}_i$  is normalized by the ones of the SA-KLT. For real vectors and transforms, we have

$$d^2(\mathbf{T}_i \underline{\mathbf{f}}_0 - \Phi^t \underline{\mathbf{f}}_0) = (\mathbf{T}_i \underline{\mathbf{f}}_0 - \Phi^t \underline{\mathbf{f}}_0)^t (\mathbf{T}_i \underline{\mathbf{f}}_0 - \Phi^t \underline{\mathbf{f}}_0) \quad (13)$$

which can be written as

$$d^2(\mathbf{T}_i \mathbf{f}_0 - \Phi' \mathbf{f}_0) = 2 \mathbf{f}_0' \mathbf{f}_0 - \mathbf{f}_0' \mathbf{T}_i' \Phi' \mathbf{f}_0 - \mathbf{f}_0' \Phi \mathbf{T}_i \mathbf{f}_0 \quad (14)$$

Since the terms in (14) are scalars, they can be written as the trace  $\text{Tr}(\cdot)$  (sum of the elements of the main diagonal of a matrix) of the  $1 \times 1$  matrix defined by the corresponding scalar. This means that

$$d^2(\mathbf{T}_i \mathbf{f}_0 - \Phi' \mathbf{f}_0) = 2 \text{Tr}(\mathbf{f}_0' \mathbf{f}_0) - \text{Tr}(\mathbf{f}_0' \mathbf{T}_i' \Phi' \mathbf{f}_0) - \text{Tr}(\mathbf{f}_0' \Phi \mathbf{T}_i \mathbf{f}_0) \quad (15)$$

Now, we can use the fact since the traces of inner and outer products are identical,  $\text{Tr}(\mathbf{f}_0' \mathbf{f}_0) = \text{Tr}(\mathbf{f}_0 \mathbf{f}_0')$ . In addition,  $\text{Tr}(\mathbf{A}') = \text{Tr}(\mathbf{A})$ . After trivial manipulations it results that

$$d^2(\mathbf{T}_i \mathbf{f}_0 - \Phi' \mathbf{f}_0) = 2 \text{Tr}(\mathbf{f}_0 \mathbf{f}_0') - 2 \text{Tr}(\mathbf{T}_i \mathbf{f}_0 \mathbf{f}_0' \Phi) \quad (16)$$

Applying the expectation operator to this expression we get  $E\{d^2(\mathbf{T}_i \mathbf{f}_0 - \Phi' \mathbf{f}_0)\} = 2 \text{Tr}(E\{\mathbf{f}_0 \mathbf{f}_0'\}) - 2 \text{Tr}(\mathbf{T}_i E\{\mathbf{f}_0 \mathbf{f}_0'\} \Phi)$  (17)

which can be expressed by

$$E\{d^2(\mathbf{T}_i \mathbf{f}_0 - \Phi' \mathbf{f}_0)\} = 2 \sum_k \lambda_k - 2 \text{Tr}(\mathbf{T}_i \mathbf{R}_{\mathbf{f}_0} \Phi) \quad (18)$$

where  $\lambda_k$  are the eigenvalues of the autocorrelation matrix of  $\mathbf{f}_0$ . To derive (18) we have used the fact that  $\text{Tr}(E\{\mathbf{f}_0 \mathbf{f}_0'\}) = \sum_k \lambda_k$  due to the energy conservation property of unitary transforms. For comparative purposes we can eliminate the constant 2 in (18). We can also normalize the metric to values between 0 and 1 by dividing the right-hand side of (18) by the sum of the eigenvalues. The final expression is given by

$$\mu = 1 - \frac{\text{Tr}(\mathbf{T}_i \mathbf{R}_{\mathbf{f}_0} \Phi)}{\sum_k \lambda_k} \quad (19)$$

For complex signals and transforms, this expression is changed to

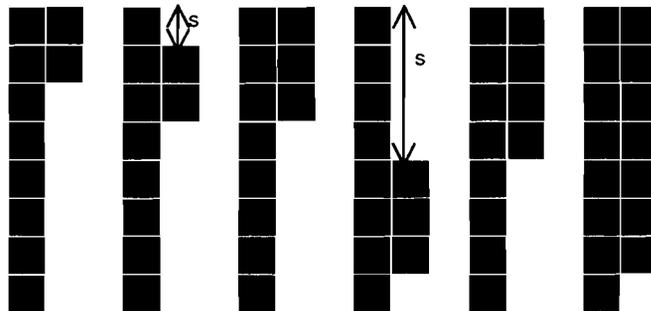
$$\mu_C = 1 - \text{Re} \left[ \frac{\text{Tr}(\mathbf{T}_i \mathbf{R}_{\mathbf{f}_0} \Phi)}{\sum_k \lambda_k} \right] \quad (20)$$

#### 4. SIMULATION RESULTS

In this section we will provide numerical results, obtained from two different SA transforms, to determine how the proposed distance measure behaves as a performance criterium to evaluate them. In our experiments, we have employed the six synthetic segments shown in Fig. 1. These segments are generated from a wide sense stationary and separable 1<sup>st</sup>-order bidimensional Markov process, with both horizontal and vertical correlation equal to 0.95.

The two different SA transforms to be compared use the 1-D DCT for the vertical and horizontal transforms and two different vertical coefficients alignment (see [1] for the mathematical formulation of generic SA transforms). One of these alignment procedures was recently proposed in [3] (*Alignment by Phase - AP*) and the other is described in [2] (*Equal Indices - EI*). These two schemes were compared in [3] using as performance criterion the cumulative energy (CE) curve. It was concluded that the method *AP* is definitely better than the *EI* approach for images that approximate a separable first order 2-D Markov process

with high correlation coefficients. This is the case of the synthetic segments depicted in Fig. 1.



(a) 8x2(s=0) (b) 8x2(s=1) (c) 8x3(s=0) (d) 8x3(s=4) (e) 8x4(s=0) (f) 8x7(s=0)

Figure 1: Synthetic segments employed in the experiments

Table I shows the results obtained from the theoretical evaluation with the metric proposed in this paper. For comparative purposes we also show in Fig. 2 the CE curves for the segments employed in the experiments. As can be seen from Table I, the nearest distances to the SA-KLT is for the method that employs the *AP* scheme. This is in accordance with the results presented in Fig. 2, where the CE curves obtained for the *AP* scheme is always larger than or equal to the ones provided by the *EI* approach. We can also verify that the segment (8x7 (s=0)) for which the difference in CE (between the *AP* and the *EI* schemes) is the smallest one is also the same segment for which the difference between the distance measure proposed in this paper is the smallest one.

The results presented in Table I corroborate the ones obtained with the CE criterion for the two SA transforms taken as examples in this paper. These results show that the proposed metric is an efficient distance measure between the SA-KLT and a given SA transform. For this reason, it can be used as an effective measure to assess the potential of a given SA transform in an object-based coding (or compression) scheme.

Table I: Results ( $\mu$  values) obtained from the theoretical evaluation

SEGMENT	METHOD	
	EQUAL INDICES	ALIGNMENT BY PHASE
8x2 ( s=0 )	0.1155	0.1034
8x2 ( s=1 )	0.1194	0.1000
8x3 ( s=0 )	0.1016	0.0875
8x3 ( s=4 )	0.1228	0.1094
8x4 ( s=0 )	0.0891	0.0827
8x7 ( s=0 )	0.1256	0.1254

Other interesting and important features concerning the proposed metric can be remarked. It may also be a profitable tool to choose the best operator that composes an SA transform. Based on the mathematical formulation described in [1], it was shown that with a simple mapping from  $R^2$  to  $R$ , we can represent any type of shape adaptive transform by means of a concatenation,  $\mathbf{T} = \mathbf{T}_3 \mathbf{T}_2 \mathbf{T}_1$ , of three linear operators. The first one ( $\mathbf{T}_1$ ) describes the one-dimensional transforms of the object columns (the vertical transforms). The second operator ( $\mathbf{T}_2$ ) represents the alignment of the vertical transform coefficients before the

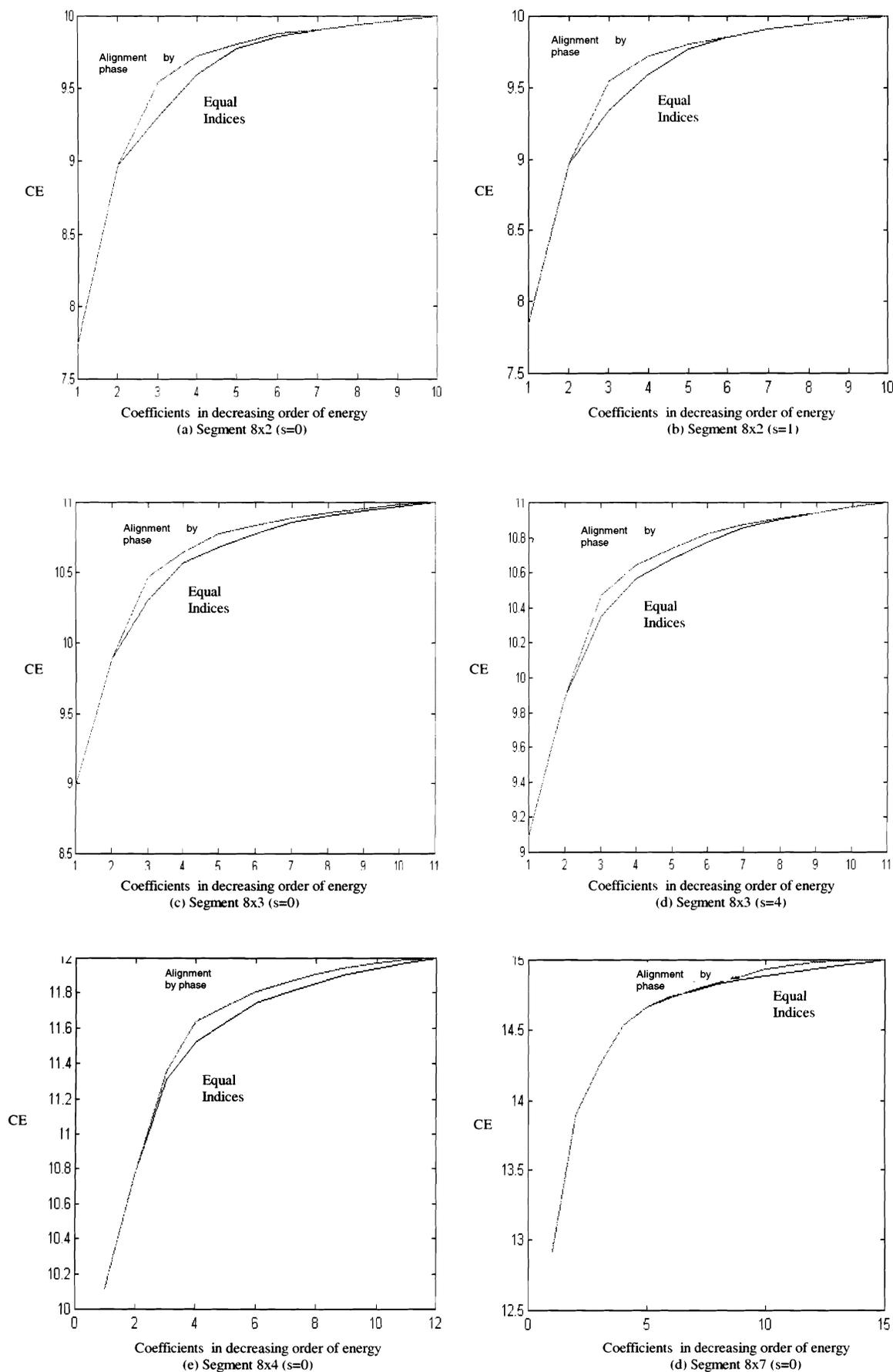


Figure 2: Cumulative energy(CE) curves for the segments employed in the experiments

application of the horizontal transforms. Finally, the third operator ( $T_3$ ) corresponds to the computation of the horizontal transforms. The simulation results presented in this paper are concerned with the performance evaluation of two possibilities for the  $T_2$  transformation, while  $T_1$  and  $T_3$  are fixed. Expressing the distance measure given by (19) in terms of these operators we get

$$\mu = 1 - \frac{\text{Tr}\left(\mathbf{T}_3 \mathbf{T}_2 \mathbf{T}_1 \mathbf{R}_{\mathbf{f}_0} \Phi\right)}{\sum_k \lambda_k} = 1 - \frac{\text{Tr}\left(\mathbf{T}_1 \mathbf{R}_{\mathbf{f}_0} \Phi \mathbf{T}_3 \mathbf{T}_2\right)}{\sum_k \lambda_k} = 1 - \text{Tr}\left(\frac{\mathbf{T}_1 \mathbf{R}_{\mathbf{f}_0} \Phi \mathbf{T}_3}{\sum_k \lambda_k} \mathbf{T}_2\right) \quad (21)$$

Note that this expression can be minimized in terms of any of the three operators. Although this is not an easy task, (21) certainly represents a mathematical tool that can be of major interest and usefulness for further research in the area of SA transforms.

## 5. DISCUSSION AND CONCLUSIONS

In this paper, we have presented a theoretical method to evaluate the performance of shape adaptive transforms. The mathematical formulation of SA transforms presented in [1] was taken as a basis for the theoretical development reported in this paper. The performance analysis of the SA-DCT and other similar transforms [2], [5]-[10] described in the literature are either based on experimental results or in terms of transform gains over PCM. They often miss the relationship to the particular mathematical structure being examined. In this sense, the approach introduced in this paper may be considered as a valuable resource to fill this gap.

A metric was proposed to assess the distance between a given SA transform and the optimum SA transform. An algebraic development was presented, and a simple expression was obtained for this metric. Simulation results show the validity of this metric as compared to experimental results previously reported in the literature [3].

Similarly to other performance criteria, such as the cumulative energy (CE) and the transform gain over PCM ( $G_{TC}$ ), the proposed metric allows the selection of the best transform among a set of several ones. Nevertheless, a major point is that this is not its sole use. Another important application is that the proposed metric may also be a profitable tool to choose the best operator that composes an SA transform.

## REFERENCE

[1] E.C. Acocella and A. Alcaim, "Mathematical formulation of shape adaptive 2D-transforms", *IEEE Signal Processing Letters*, vol. 8, no. 10, p. 273-275, Oct. 2001.  
[2] T. Sikora and B. Makai, "Shape-adaptive DCT for generic coding of video", *IEEE Trans. Circuits Syst. Video Technol.*, vol. 5, no. 1, p. 59-62, Feb. 1995.  
[3] E.C. Acocella and A. Alcaim, "Alignment by phase of vertical coefficients in SA-DCT", *IEEE Signal Processing Letters*, vol. 8, no. 2, p. 42-44, Feb. 2001.  
[4] A.K. Jain, *Fundamentals of digital image processing*, Englewood Cliffs, Prentice-Hall, Inc., 1989.

[5] A. Kaup, "Object-based texture coding of moving video in MPEG-4", *IEEE Trans. Circuits Syst. Video Tech.*, vol. 9, no. 1, p. 5-15, Feb. 1999.  
[6] T. Sikora, S. Bauer and B. Makai, "Efficiency of shape-adaptive 2-D transforms for coding of arbitrarily shaped image segments", *IEEE Trans. Circuits Syst. Video Technol.*, vol. 5, no. 3, pp. 254-258, June 1995.  
[7] C. Stiller and J. Konrad, "Region-adaptive transform based on stochastic model", *Proc. IEEE Int. Conf. on Image Processing*, 1995, p. 264-267.  
[8] M. Gilge, T. Engelhardt and R. Mehlan, "Coding of arbitrarily shaped image segments based on a generalized orthogonal transform", *Signal Processing: Image Communication*, vol. 1, p. 153-180, 1989.  
[9] M. Bi, S. H. Ong and Y. H. Ang, "Comment on 'Shape Adaptive DCT for generic coding of video'", *IEEE Trans. Circuits Syst. Video Tech.*, vol. 6, no. 6, p. 686-688, Dec. 1996.  
[10] T. Xie, C. Weng, Y. He, and C. Feng, "A new approach to arbitrary shaped DCT", *Proc. PCS 97*, p. 97-101, Sep. 1997.  
[11] J. M. F. Moura and M. G. S. Bruno, "DCT/DST and Gauss-Markov Fields: Conditions for Equivalence", *IEEE Transactions on Signal Processing*, vol. 46, no. 9, p. 2571-2574, Sep. 1998.  
[12] J. M. F. Moura and N. Balram, "Recursive structure of noncausal Gauss-Markov random fields", *IEEE Transactions on Information Theory*, vol. 38, p. 334-354, March 1992.  
[13] M. Püschel and J. M. F. Moura, "The Discrete Trigonometric Transforms and their Fast Algorithms: An Algebraic Symmetry Perspective", *Proceedings of the 10th Digital Signal Processing Workshop*, 2002.

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**Emilio Carlos Acocella and Abraham Alcaim**  
**A Distance Measure for Performance Evaluation of Shape Adaptive Transforms**

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