Coded Multi-dimensional Spreading System using the Discrete Fourier Transform

Haesik Kim, Garik Markarian, *Member, IEEE,* and Valdemar C. da Rocha Jr., *Senior Member, IEEE.*

***Abstract*—In this paper a novel coded multi-dimensional spreading technique using the Discrete Fourier Transform (DFT) is introduced. It exploits the DFT characteristic of spreading one symbol in one domain to all the symbols in the corresponding transform domain. Frequency domain channel estimation is performed at the receiver. The performance of a coded two-dimensional (2D) DFT spreading communication system is investigated and simulation results are presented, using first a 2D DFT time-frequency system and then a 2D DFT space-frequency system. The proposed 2D DFT system shows good immunity to narrowband interference and impulse noise and exhibits a significant performance improvement when compared with a coded conventional OFDM system of equivalent size.**

***Index Terms*—Discrete Fourier Transform, Coded systems, OFDM, Multi-dimensional spreading.**

I. INTRODUCTION

HIGH speed communication systems such as ADSL [1], IEEE 802.16 [2], IEEE 802.11 [3], to mention a few cases, have adopted the discrete Fourier transform (DFT) [4], [5] for multi-carrier transmission. Each point of a DFT acts like a subcarrier in a multi-carrier communication system. In practice, it is much easier to generate subcarriers using the DFT rather than conventional local oscillators. However, there is one more important characteristic of the DFT which is to spread one point to a whole block of points. We notice that although orthogonal frequency division multiplexing (OFDM) systems [6], [7] basically use the DFT for multi-carrier transmission, they also simultaneously spread one symbol to a whole block of points. In this manner OFDM systems naturally provide a diversity gain. The one-dimensional spreading technique is widely used in modern communication systems because it allows multiple-access and offers diversity gain. For example, code division multiple access (CDMA) systems [8] assign specific codes such as a direct sequence code or a frequency hopping code [8] to each mobile station and each base station for sharing a limited resource. In case the spreading technique is used for diversity gain, a distorted symbol can still be recovered from the other non-distorted remaining symbols.The 2-dimensional fast Fourier Transform (2D FFT) technique [4] is not a new technique and is widely used in the area of image signal processing [11]. However its application as a modulation technique is fairly recent and has been investigated in [16], where an uncoded version of it has been shown to performs well under jamming and impulse noise environments. In [17] an uncoded 2D FFT application for relay communications has been investigated.

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In this paper we propose a coded multi-dimensional spreading system using the multi-dimensional DFT and demonstrate its usefulness by means of examples. A coded 2-dimensional time-frequency spreading system, as the first example, is constructed and evaluated following WiMAX specifications and a 2-dimensional space-frequency spreading system is constructed as the second example. As a particular case, the performance of a coded two-dimensional DFT (2D DFT) spreading system is evaluated and compared to that of a conventional 1-dimensional spreading system based on OFDM. In Section II the multi-dimensional DFT is reviewed and its spreading properties are illustrated for the 2-dimensional case. In Section III we propose a coded two-dimensional time-frequency spreading [9] and in Section IV we propose and discuss a coded 2-dimensional space-frequency spreading system. It Section V computer simulation results are presented and the paper ends in Section VI with closing remarks.

II. THE MULTI-DIMENSIONAL DISCRETE FOURIER TRANSFORM

The conventional 1-dimensional DFT, which is described by a single variable, admits a natural generalisation in more dimensions [10]. The multi-dimensional DFT has an associated representation by a multi-dimensional array such that for a d-dimensional DFT, one element of the corresponding array is denoted as $x_{n_1,n_2,\ldots,n_d}$, with discrete variables $n_1, n_2, \ldots, n_d$, $0 \leq n_\ell \leq N_\ell - 1$, where N_ℓ denotes a positive integer and $1 \leq \ell \leq d$. The multidimensional DFT [10] having d dimensions is defined as follows.

$$X_{k_1,k_2,\ldots,k_d} = \sum_{n_1=0}^{N_1-1}\left(\omega_1^{k_1 n_1}\sum_{n_2=0}^{N_2-1}\left(\omega_2^{k_2 n_2}\ldots \ldots\sum_{n_d=0}^{N_d-1}\omega_d^{k_d n_d}x_{n_1,n_2,\ldots,n_d}\right)\ldots\right), \quad (1)$$

where $\omega_\ell = e^{-\frac{2\pi i}{N_\ell}}$, $0 \leq k_\ell \leq N_\ell - 1$, $1 \leq \ell \leq d$.

It follows from Equation (1) that each element $x_{n_1,n_2,\ldots,n_d}$ is transformed by the factor $\omega_\ell^{k_\ell n_\ell}$, $1 \leq \ell \leq d$, and after the appropriate summations are performed it produces the transformed multidimensional array with elements denoted as $X_{k_1,k_2,\ldots,k_d}$ in a different domain. Similar to the 1-dimensional DFT case, the multi-dimensional inverse DFT having d dimensions is defined as follows [10].

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$$x_{n_1,n_2,\ldots,n_d} = \frac{1}{\prod_{\ell=1}^{d} N_\ell} \sum_{k_1=0}^{N_1-1} \left(\omega_1^{-k_1 n_1} \sum_{k_2=0}^{N_2-1} \left(\omega_2^{-k_2 n_2} \cdots \cdots \sum_{k_d=0}^{-N_d-1} \omega_d^{-k_d n_d} X_{k_1,k_2,\ldots,k_d} \right) \cdots \right). \quad (2)$$

The calculation process in (2) is analogous to that for calculating the multidimensional DFT in (1). There is however a more compact and elegant manner to represent the multidimensional DFT using vectors.

Let $\mathbf{n} = (n_1, n_2, \ldots, n_d)$ and let $\mathbf{k} = (k_1, k_2, \ldots, k_d)$ denote d-dimensional vectors, $\mathbf{0} \le \mathbf{n} \le \mathbf{N}-1$, where $\mathbf{0} \triangleq (0,0,\ldots,0)$, $\mathbf{N}-1 \triangleq (N_1-1, N_2-1, \ldots, N_d-1)$. When written in vectorial notation, Equation (1) is, equivalently, expressed as follows.

$$X_{\mathbf{k}} = \sum_{\mathbf{n}=0}^{\mathbf{N}-1} e^{-2\pi i \mathbf{k}\cdot(\mathbf{n}/\mathbf{N})} x_{\mathbf{n}}\,, \quad (3)$$

where $\mathbf{n}/\mathbf{N} \triangleq (n_1/N_1, \ldots, n_d/N_d)$ to be performed element-wise, and the sum denotes the set of nested summations as seen earlier in (1).

Applying the vectorial notation to Equation (2) the inverse of the d-dimensional DFT is expressed as follows.

$$x_{\mathbf{n}} = \frac{1}{\prod_{\ell=1}^{d} N_\ell} \sum_{\mathbf{k}=0}^{\mathbf{N}-1} e^{2\pi i \mathbf{n}\cdot(\mathbf{k}/\mathbf{N})} X_{\mathbf{k}}\,. \quad (4)$$

In spite of an apparently more involved formulation, the multidimensional DFT is amenable to a simple interpretation. While the one-dimensional DFT (1D DFT) expresses an input x_n as a superposition of sinusoids, the multidimensional DFT expresses its input as a superposition of plane waves, or sinusoids oscillating in space along the directions indicated by $\mathbf{k}/\mathbf{N}$ and having amplitudes defined by $X_{\mathbf{k}}$. Such a decomposition turned out of great importance in practice, for example, in digital image processing $(d = 2)$ [11, pp.81-125] or for solving partial differential equations in three or more dimensions $(d \ge 3)$ by the spectral method, i.e., a method by which a linear differential equation is transformed into an ordinary algebraic equation, easily solved. In computational terms the multidimensional DFT can be interpreted as resulting from the composition of a sequence of 1D DFTs along each dimension.

A. A 2-dimensional DFT Spreading System

In the two-dimensional (2D) case, i.e., where $\mathbf{n} = (n_1, n_2)$, from $x_{\mathbf{n}}$ one can first compute $y_{\mathbf{n}'}$, which denotes N_1 independent 1D DFTs of size N_2 along n_2 (call them rows) to form a new array, i.e.,

$$y_{\mathbf{n}'} = \sum_{n_2=0}^{N_2-1} x_{\mathbf{n}} \omega_2^{k_2 n_2},$$

where $\mathbf{n}' = (n_1, k_2)$, and then compute $X_{\mathbf{k}}$, which denotes N_2 independent 1D DFTs of size N_1 along n_1 (call them columns), to form the final result , i.e.,

$$X_{\mathbf{k}} = \sum_{n_1=0}^{N_1-1} y_{\mathbf{n}'} \omega_2^{k_1 n_1},$$

where $\mathbf{k} = (k_1, k_2)$. Since the nested summations in Equation (1) commute, one can alternately transform first the columns and then the rows. It follows from this commuting property of the multidimensional DFT that, once an efficient way is given to compute a 1D DFT (e.g., an ordinary one-dimensional FFT algorithm), one immediately has a way to efficiently compute the multidimensional DFT. In the two-dimensional case this is known as a row-column algorithm, although there are also intrinsically multidimensional FFT algorithms available in the literature [10], [12]. Obviously, the 2D DFT is just a particular case of the multidimensional DFT and an analogous argument for computing a d-dimensional DFT follows unchanged.

At this point we would like to call the reader's attention to the spreading effect of the multidimensional DFT, as indicated for example in (3), in the sense that a given value of $x_{\mathbf{n}}$, which we may call a symbol in the $\mathbf{n}$-domain, is spread by means of the multidimensional DFT along each dimension of the corresponding transform domain, or $\mathbf{k}$-domain. In multidimensional spreading, a dimension can designate time, or frequency, or space, etc. As it is widely known, a diversity scheme is a method for improving the reliability of a signal by utilizing two or more communication channels with different characteristics. This is a very important technique in modern telecommunications to compensate for multipath fading and interference, thus combating burst errors. Diversity is based on the fact that individual channels experience different levels of fading and interference. Therefore, we can obtain more diversity gain if sending one symbol through a multi-dimensional channel because the corresponding diversity gain will increase if the number of dimensions is increased.

As an application of a 2D spreading system, we propose in Section IV a space-frequency system, where an element of the data 2D array spreads into the space and the frequency domains. The space dimensional spreading size is given by the number of antennas. A space-frequency spreading system could be useful for relay communication [19] or sensor network [20] which can have many antennas. In particular, it is more useful for a condition where the signal strengths for each relay (or sensor) antenna are different and where noise and interference in independent wireless channels are different.

III. Coded 2D Time-Frequency Spreading System

In this section we introduce a coded 2D time-frequency spreading system. The coded 2D time-frequency spreading transmitter is illustrated in Figure 1, where a_{ij} denotes a symbol at the input to the turbo product code (TPC) encoder [19]. The TPC is based on the product of two component codes which are used in a 2-dimensional matrix form [13]. The k_x information bits in the rows are encoded into n_x bits using the component (n_x, k_x) block code specified for the composite code. After encoding the rows, the columns are encoded using a block code (n_y, k_y), where the check bits of the first code are also encoded. The overall block size n of such a product code is $n = n_x n_y$, the total number k of information bits is $k = k_x k_y$ and the code rate R is $R = R_x R_y$, where $R_x = k_x/n_x$ and $R_y = k_y/n_y$.

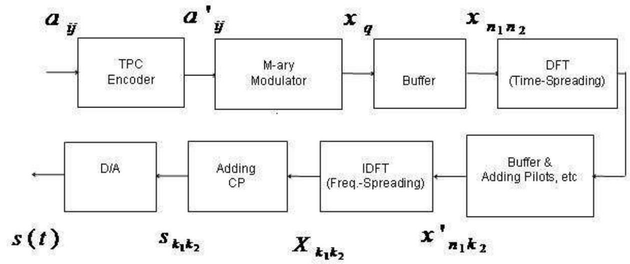

Fig. 1. 2-dimensional (Time-Frequency) spreading transmitter system.

Assuming that x_q represents a complex-valued signal point in an M-ary constellation, for example, assuming it is $x_q = a'_{ij} + i a'_{i+1,j}$ if QPSK modulation is employed. After modulation, the x_q values are fed to a buffer of size N_2 which stores the incoming x_q symbols, forming blocks of N_2 of them which is the 2D DFT size in the column direction. Therefore, $x_{n_1 n_2}$ enters DFT block and one element of $x_{n_1 n_2}$ spreads to a whole column as follows.

$$x'_{n_1 k_2} = \sum_{n_2=0}^{N_2-1} \omega_2^{k_2 n_2} x_{n_1 n_2},$$

where $0 \le k_2 \le N_2 - 1$ and $0 \le n_1 \le N - 1$.

On the next step, we store the DFT symbol into the buffer in a column by column basis as illustrated in Figure 2 and then the pilots, DC, and guard interval are inserted along a column direction. Therefore, we compute an IDFT of size N_1 in the row direction, having for input the symbols $x'_{n_1 k_2}$, in a row by row basis. The IDFT output, denoted as $X_{k_1 k_2}$, is the time domain OFDM symbol expressed as follows.

$$X_{k_1 k_2} = \frac{1}{N_1} \sum_{n_1=0}^{N_1-1} \omega_1^{-k_1 n_1} x'_{n_1 k_2},$$

where $0 \le k_1 \le N_1 - 1$ and $0 \le k_2 \le N_2 - 1$.

After adding to each row a cyclic prefix containing N_g samples we get,

$$s_{k_1 k_2} = \frac{1}{N_s} \sum_{n_1=0}^{N_s-1} x'_{n_1 k_2} \omega_1^{-k_1 n_1},$$

where $N_s = N_1 + N_g$, $-N_g \le k_1 \le N_1 - 1$ and $0 \le k_2 \le N_2 - 1$. The signal $s_{k_1 k_2}$ enters the digital to analogue (D/A) converter to provide the transmit signal $s(t)$.

A 2D time-frequency spreading receiver is illustrated in Figure 3. After the analogue to digital (A/D) conversion, the cyclic prefix is removed in the received signal $r_{k_1 k_2}$ and then the resulting signal $Y_{k_1 k_2}$ enters the DFT block, the output of which is $y'_{n_1 k_2}$ and is expressed as follows.

$$y'_{n_1 k_2} = \sum_{k_1=0}^{N_1-1} Y_{k_1 k_2} \omega_1^{k_1 n_1},$$

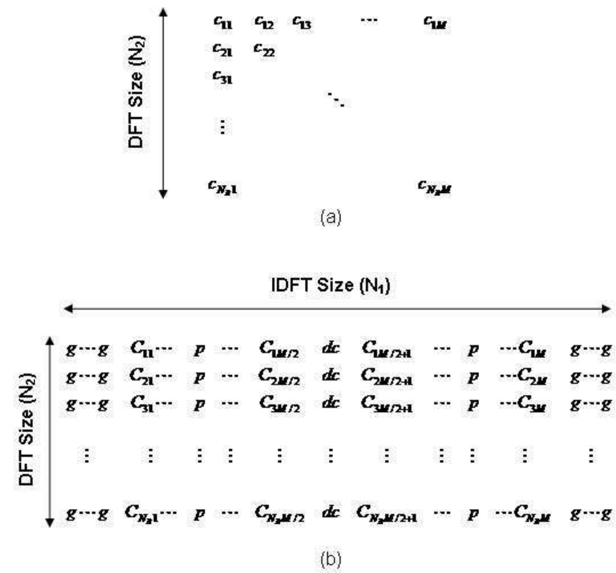

Fig. 2. First buffer block structure (a) and second buffer block structure in the transmitter.

where $0 \le n_1 \le N_1 - 1$ and $0 \le k_2 \le N_2 - 1$. On the next step, we store $y'_{n_1 k_2}$ into the buffer in a row by row manner and then carry out an estimation of the channel frequency response $H_{n_1 k_2}$ based on least squares (LS) estimation (please refer to Appendix [21]) which is given by

$$H_{n_1 k_2} = \frac{Y^p_{n_1 k_2}}{X^p_{n_1 k_2}},$$

where $X^p_{n_1 k_2}$ denotes block type pilots we already know and $Y^p_{n_1 k_2}$ denotes block type pilots we received. Therefore, we can compensate the OFDM symbol with channel impairment by using $y''_{n_1 k_2}$ instead of $y'_{n_1 k_2}$ as follows.

$$y''_{n_1 k_2} = H_{n_1 k_2} y'_{n_1 k_2}.$$

After channel estimation, we store $y''_{n_1 k_2}$ into the buffer until it is filled with the IDFT of column size (N_2) and we then the symbol in the buffer is output in a column by column manner. Therefore, $y''_{n_1 k_2}$ enters the IDFT block to produce $y_{n_1 n_2}$ as follows.

$$y_{n_1 n_2} = \frac{1}{N_2} \sum_{k_2=0}^{N_2-1} y''_{n_1 k_2} \omega_2^{-k_2 n_2},$$

for $0 \le n_2 \le N_2 - 1$ and $0 \le n_1 \le N_1 - 1$.

After obtaining $y_{n_1 n_2}$ with the IDFT, we carry out demodulation, i.e., $y_{n_1 n_2} = b'_{ij} + i b'_{i+1,j}$ if it is QPSK demodulation. The coded bit b'_{ij} is then obtained. The coded bit b'_{ij} enters the TPC decoder, and b_{ij} is obtained after the TPC decoding.

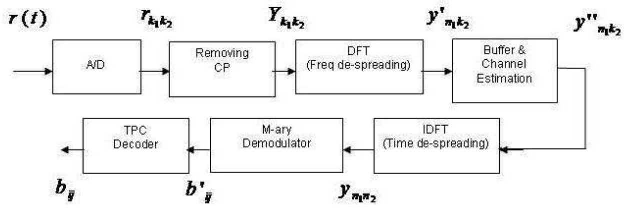

Fig. 3. 2-dimensional spreading (Time-Frequency) receiver system.

A. Spreading Effect

1) Signal Model: From the receiver process, $y_{n_1n_2}$ is written as follows.

$$\begin{aligned} y_{n_1n_2} &= \frac{1}{N_2}\sum_{k_2=0}^{N_2-1} y''_{n_1k_2}\omega_2^{-k_2n_2} \\ &= \frac{1}{N_2}\sum_{k_2=0}^{N_2-1} H_{n_1k_2}y'_{n_1k_2}\omega_2^{-k_2n_2} \\ &= \frac{1}{N_2}\sum_{k_2=0}^{N_2-1} H_{n_1k_2}\left(\sum_{k_1=0}^{N_1-1} Y_{k_1k_2}\omega_1^{k_1n_1}\right)\omega_2^{-k_2n_2} \\ &= \frac{1}{N_2}\sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} H_{n_1k_2}Y_{k_1k_2}\omega_1^{k_1n_1}\omega_2^{-k_2n_2}, \end{aligned}$$

for $0 \le n_2 \le N_2-1$ and $0 \le n_1 \le N_1-1$.

2) Narrowband Interference and Doppler Effect: Narrowband interference and Doppler effect are considered within the transmission bandwidth, assuming that the timing offset is smaller than the guard interval and that no intersymbol interference (ISI) occurs. The received signal, denoted by $r_{n_1n_2}$, with the cyclic prefix removed and including narrowband interference and frequency offset caused by the Doppler effect is given by

$$r_{n_1n_2} = y_{n_1n_2}e^{2\pi f_d} + I_{k'_1} + n_0,$$

where $f_d, I_{k'_1}$ and n_0 denote frequency offset, narrowband interference and additive white Gaussian noise (AWGN) sample, respectively. Narrowband interference within transmission bandwidth is expressed as follows.

$$I_{k'_1} = \sum_{n'_1=0}^{N_{nb}-1} i_{n'_1}e^{\frac{2\pi i}{N_{nb}}n'_1k'_1},\ 0 \le k'_1 \le N_{nb}-1,$$

where N_{nb} and $i_{n'_1}$ denote the number of interfering signals with same subcarrier bandwidth within the transmission bandwidth N_1 and interference amplitude respectively. Equivalently, the received signal $r_{n_1n_2}$ is expressed as follows.

$$\begin{aligned} r_{n_1n_2} &= \frac{1}{N_2}\sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} H_{n_1k_2}Y_{k_1k_2}\omega_1^{k_1n_1}\omega_2^{-k_2n_2}e^{2\pi f_d} \\ &+ \sum_{n'_1=0}^{N_{nb}-1} i_{n'_1}e^{\frac{2\pi i}{N_{nb}}n'_1k'_1} + N_0, \end{aligned} \tag{5}$$

for $0 \le n_2 \le N_2-1, 0 \le n_1 \le N_1-1$ and $0 \le k'_1 \le N_{nb}-1$. It is noticed in Equation (5), that the received signal $Y_{k_1k_2}$ is spread due to the action of the time domain term $\omega_1^{k_1n_1}$ and the frequency domain term $\omega_2^{-k_2n_2}$. Likewise the effect of narrowband interference and frequency offset is also spread in the frequency domain and in the time domain. In this manner diversity gain is obtained from the time domain and the frequency domain.

IV. A Coded 2D DFT Space-Frequency Spreading System

We propose a coded 2D space-frequency spreading system, having one frequency dimension and one space dimension, based on a MIMO-OFDM system [15]. The coded 2D space-frequency spreading transmitter is illustrated in Figure 4. It is similar to the coded 2D time-frequency spreading transmitter except that it employs a space-time mapping block. After column spreading, each symbol $z_{n_1k_2}$, $0 \le k_2 \le N_2-1$, is mapped into a space-time mapping block $x_{n_1k_2}^{s_\ell}$, $0 \le \ell \le N_2-1$, i.e.,

$$x_{n_1k_2}^{s_\ell} = M_\ell(z_{n_1k_2}),\quad 0 \le \ell \le N_2-1,$$

where N_2 denotes the number of antennas, i.e., the spreading size, and M_ℓ denotes the mapping function. The 2-dimensional space-frequency spreading receiver is illustrated in Figure 5. It is also similar to a 2-dimensional time-frequency spreading receiver except for the space-time de-mapping and combining block. After row de-spreading, each symbol is de-mapped and combined, which we denote as follows.

$$y_{n_1k_2}^{s_\ell} = D_\ell(y''_{n_1k_2}),\quad 0 \le \ell \le N_2-1,$$

where N_2 denotes the number of antennas and D_ℓ denotes the de-mapping and combining function.

We consider next a two-input single-output (2x1) channel where a space-time block code [18] is used and z^* denotes the complex conjugate of z. Each symbol $z_{n_1k_2}$, $0 \le k_2 \le 1$, is mapped into a space-time mapping block $x_{n_1k_2}^{s_\ell}$, $0 \le \ell \le 1$, as follows.

$$\begin{bmatrix} x_{n_10}^{s_0} \\ x_{n_11}^{s_0} \end{bmatrix} = \begin{bmatrix} z_{n_10} \\ -z_{n_11}^* \end{bmatrix} \quad \text{for antenna 1.}$$

$$\begin{bmatrix} x_{n_10}^{s_1} \\ x_{n_11}^{s_1} \end{bmatrix} = \begin{bmatrix} z_{n_11} \\ z_{n_10}^* \end{bmatrix} \quad \text{for antenna 2.}$$

The data symbols x_{n_10} and x_{n_11} spread each to both z_{n_10} and z_{n_11}. The latter are encoded by space-time coding, and thus spread to each antenna. A received symbol at the output of the channel estimation block in Figure 7, represented by the pair (y''_{n_10}, y''_{n_11}), can be expressed as follows.

$$\begin{aligned} y''_{n_10} &= h_0z_{n_10} + h_1z_{n_11} + n_0 \\ y''_{n_11} &= -h_0z_{n_11}^* + h_1z_{n_10}^* + n_1, \end{aligned} \tag{6}$$

where h_i and n_i denote, respectively, the channel complex multiplicative distortion and a complex random variable representing receiver noise and interference for antenna $i, 0 \le i \le 1$. A received symbol, represented by the pair $(y_{n_10}^s, y_{n_11}^s)$, after space-time de-mapping can be combined as follows.

TABLE I
SIMULATION CONFIGURATION FOR 2D CODED TIME-FREQUENCY SPREADING SYSTEM

Error-Correcting Coding	Turbo product code: 23 bytes (data block size) 48 bytes (coded block size) Component codes $(32, 26, 4), (16, 11, 4)$ Code parameters $\ell_x = 4, \ell_y = 2,$ $B = 8, Q = 6$ Decoder soft quantising: 4 bits Number of iterations: 3
OFDM parameters	QPSK modulation DFT size: 256 Cyclic prefix: 1/4 Channel bandwidth: 20MHz
Channel	Multipath Rayleigh fading: 4 delay vector Doppler shift: 50Hz, 70Hz AWGN Interference: 1.49MHz within WiMAX bandwidth
Channel estimation	Freq. domain: block-type LS estimation
Packet size	256×64 (IDFT size $\times$ DFT size in the transmitter)

TABLE II
SIMULATION CONFIGURATION FOR 2D CODED SPACE-FREQUENCY SPREADING SYSTEM

Error-Correcting Coding and MIMO	Convolutional code of rate 1/2 Space-time block coding
OFDM parameters	QPSK modulation DFT size: 1024 Cyclic prefix: 1/8 Channel bandwidth: 10MHz
Channel	MISO (2×1) channel Multipath Rayleigh fading: 6 delay vector Doppler shift: 70Hz AWGN Interference: 1.49MHz and 2.98MHz
Channel estimation	Freq. domain: block-type LS estimation
Packet size	1024×2 (IDFT size $\times$ DFT size in the transmitter)

$$\begin{bmatrix} y^s_{n_1 0} \\ y^s_{n_1 1} \end{bmatrix} = \begin{bmatrix} h_0^* y''_{n_1 0} + h_1 (y''_{n_1 1})^* \\ h_1^* y''_{n_1 0} - h_0 (y''_{n_1 1})^* \end{bmatrix}. \quad (7)$$

Substituting (6) into (7), the pair $(y^s_{n_1 0}, y^s_{n_1 1})$ can be expressed as follows.

$$\begin{bmatrix} y^s_{n_1 0} \\ y^s_{n_1 1} \end{bmatrix} = \begin{bmatrix} (|h_0|^2 + |h_1|^2) y''_{n_1 0} + h_0^* n_0 + h_1 n_1^* \\ (|h_0|^2 + |h_1|^2) y''_{n_1 1} - h_0 n_1^* + h_1^* n_0 \end{bmatrix}. \quad (8)$$

V. COMPUTER SIMULATION RESULTS

A computer simulation has been carried out on a 2D DFT spreading system following WiMAX specifications. It is remarked that if the DFT block in the transmitter is removed as well as the IDFT block in the receiver, the resulting structure is the same as the WiMAX physical layer structure. The main parameters employed in the simulation are given in Table I and in Table II.

Figures 6 and 7 show that the bit error rate performance of the proposed 2D coded spreading system, under Doppler effect and narrowband interference, is better than that of a 1D spreading system (conventional OFDM system) of same size, and presents lower error-floor. For the OFDM in the first simulation, a symbol encoded by a turbo product code and modulated by a QPSK has packet size 256x64. Namely, one symbol is spread to 256 subcarriers in the frequency domain and to 64 time slots in the time domain. In the second simulation, a symbol is also encoded and has packet size 1024x2, i.e., one symbol is spread to 1024 subcarriers in the frequency domain and to two antennas in space domain.

Figure 8 indicates the the proposed 2D coded spreading system has a slightly better performance than a conventional 1D spreading system with a space-time block code. This performance gain difference between a 2D Time-Frequency spreading system and a 2D space-frequency spreading system is caused by the spreading size. The spreading size for a 2D space-frequency spreading system is only 2 because it equals the number of transmitting antennas.

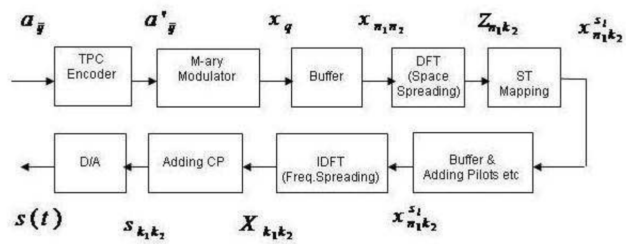

Fig. 4. 2-dimensional space-frequency spreading transmitter system.

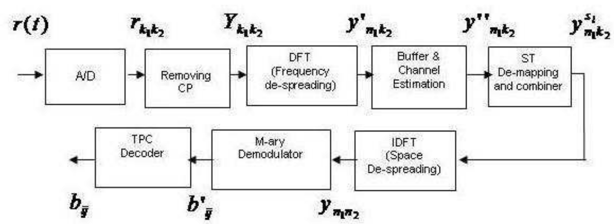

Fig. 5. 2-dimensional space-frequency spreading receiver system.

VI. CLOSING REMARKS

We have introduced a coded multi-dimensional spreading system using the discrete Fourier transform and shown by example that the 2D coded spreading system has better performance than the 1D spreading system of the same size, under Doppler effect and narrowband interference. In Section V simulation results of a 2D DFT coded spreading system were presented. We notice that a multi-dimensional DFT spreading system has higher diversity gain than a 1D DFT spreading system. The weak point of the proposed spreading system is that the multi-dimensional spreading system requires a bigger packet size and more DFT blocks are needed so that latency and complexity are increased. The latency and complexity will be increased with increasing spreading size and number of DFT blocks, respectively. If one or more symbols are affected by narrowband interference, deep fading or the Doppler effect, they can still be recovered from the remaining (unaffected) symbols because one symbol is spread to multiple dimensions.

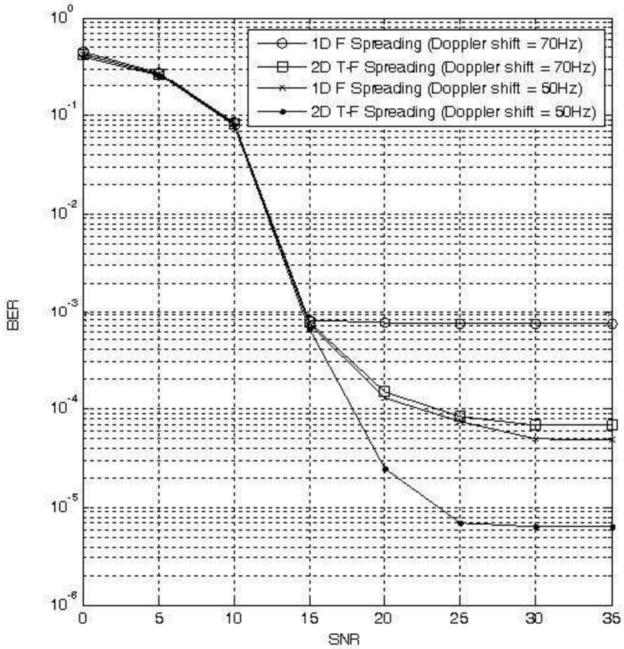

Fig. 6. Comparison of 2-D time-frequency spreading system with 1-D frequency spreading system under the Doppler effect.

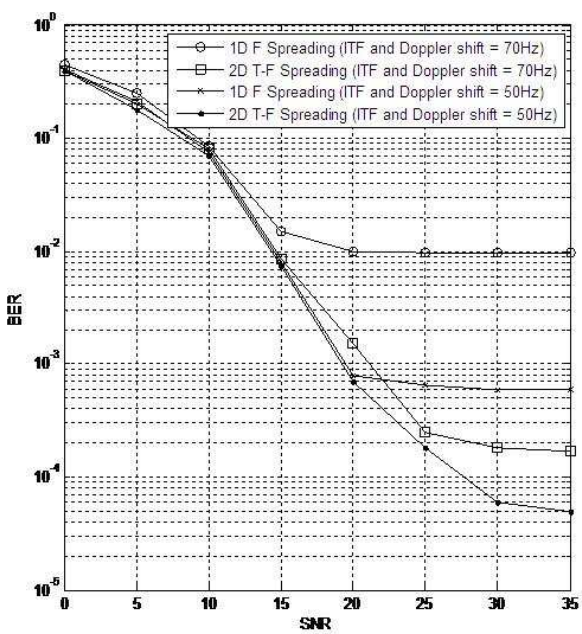

Fig. 7. Comparison of 2-D time-frequency spreading system with 1-D frequency spreading system under the Doppler effect and narrow band interference.

In the case of a 2D time-frequency (or space-frequency) DFT coded spreading system, it spreads not only in the frequency domain but also in the time (or space) domain. In order to compensate for possible error bursts, an interleaver can be used but it is not the same as spreading as meant here.

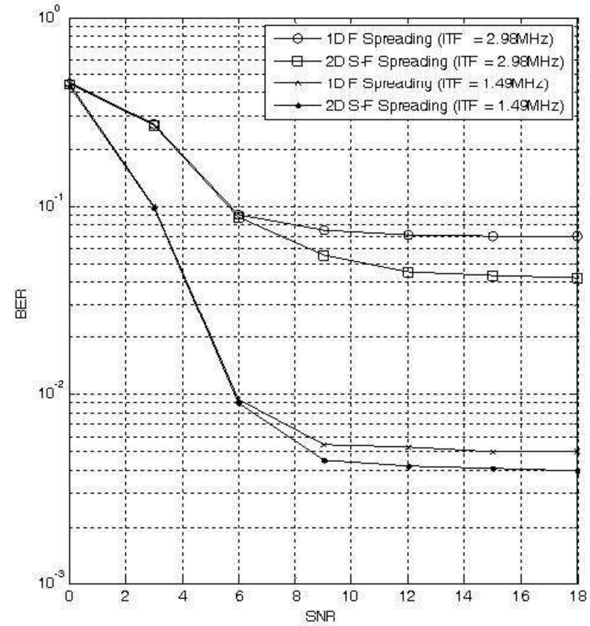

Fig. 8. Comparison of 2-D space-frequency spreading system with 1-D frequency spreading system under narrowband interference.

An interleaver changes error locations but effectively does not reduce the number of errors. Finally, a possible practical application of coded multi-dimensional DFT spreading can be in the form of a coded 2D DFT spreading system, applied to the next generation of WiMAX systems because the 2D DFT spreading architecture is based on the WiMAX architecture.

Appendix A
A Compact Description of the OFDM Technique

An OFDM system converts a serial data stream into parallel blocks of size N and modulates these blocks using the inverse fast Fourier transform (IFFT). Time domain samples $x(n)$ of an OFDM symbol are obtained from frequency domain data symbols $X(k)$ as follows.

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j2\pi nk/N}, \ 0 \leq n \leq N-1,$$

where $X(k)$ denotes the data symbol transmitted by the kth subcarrier of the OFDM system, N denotes the fast Fourier transform (FFT) size, and K denotes the set of OFDM subcarriers available for transmission. After the addition of a cyclic prefix (CP) and digital to analogue conversion, the signal is sent through the mobile radio channel. The channel is usually assumed to be constant over an OFDM symbol, but time-varying across OFDM symbols, which is a reasonable assumption for low and medium mobility. At the receiver, the signal contaminated by noise is received. After synchronization, down-sampling, and removal of the CP, the simplified baseband model of the received samples can be represented as follows.

$$y(n) = \sum_{\ell=0}^{L-1} x(n-l)h(l) + w(n),$$

where L denotes the number of sample-spaced channel taps, $w(n)$ denotes additive white Gaussian noise (AWGN) samples with zero mean and variance of σ_w^2, and the time domain channel impulse response for the current OFDM symbol, $h(\ell)$, is represented as a time-invariant linear filter. In this case, after calculating the FFT of the received signal $y(n)$, the samples in frequency domain can be written as follows.

$$Y(k) = X(k)H(k) + W(k), \quad k \in K, \tag{9}$$

where H and W denote the FFTs of h and w, respectively.

APPENDIX B
LEAST SQUARES CHANNEL ESTIMATION

The least squares (LS) estimate of the channel frequency response $\mathbf{H}$ [21] can be calculated using the received signal and the knowledge of transmitted symbols as

$$\hat{H}_{LS} = \frac{Y(k)}{X(k)} = H(k) + \frac{W(k)}{X(k)}.$$

The LS method is the simplest channel estimation method and it is usually used as an initial step for more advanced algorithms.

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