D-ARM : A NEW PROPOSAL FOR MULTI-DIMENSIONAL INTERCONNECTION NETWORKS

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Abstract - This paper presents a new topology for multidimensional interconnection networks, namely D-ARM, which has the goal of simultaneously providing high network transmission capacity and a low information transfer delay. The new D-ARM topology has a connection pattern arranged in *alternated regular mesh* fashions with toroidal boundaries. Five distinct network attributes, normally used to characterize interconnection network topologies, were employed to analyze the D-ARM topology: the network diameter, bisection width, deflection index, degree of connectivity and symmetry. Also the evaluation of the performance of the D-ARM network through computer simulations was carried out based on the following measures: throughput and information transfer delay. An upper-bound to the network transmission capacity was derived in function of the network dimension (D) and length (W). In order to validate our proposal, as a viable topology among other well-known topologies, a comparative analysis among D-ARM, MSN and ShuffleNet was performed. The analysis results show that the D-ARM outperforms MSN and ShuffleNet in many aspects and suggest some plausible applications of the D-ARM networks, e.g., broadband switching architectures, multiprocessor connection, high-speed MAN, WDM optical networks and photonic networks.

Resumo - Este artigo apresenta uma nova topologia para redes de interconexão multidimensionais, denominada D-ARM, que tem como objetivo alcançar simultaneamente uma alta capacidade de transmissão e um atraso de transferência reduzido. A nova topologia D-ARM possui um padrão de conexões arranjadas na forma de malha regular e alternadas com fronteiras toroidais. Cinco características da rede. normalmente usadas para caracterizar topologias de redes de interconexão foram empregadas na análise da topologia D-ARM : diâmetro, largura da bisecção, indice de deflexão, grau de conectividade e simetria. A avaliação do desempenho da rede D-ARM também foi feita via simulação computacional baseada nas seguintes medidas : vazão e atraso de transferência de informações. Um limite superior para a capacidade de transmissão da rede foi derivada em função da dimensão (D) e do comprimento (W) da rede. Para convalidar nossa proposta, como uma topologia viável entre outras topologias conhecidas, uma análise comparativa entre D-ARM, MSN e ShuffleNet foi realizada. Os resultados da análise mostram que a D-ARM tem desempenho melhor que MSN e ShuffleNet sob vários aspectos e pode ser empregada em várias aplicações, tais como, arquitetura de chaveamento em faixa larga, conexão de multiprocessadores, MAN de alta velocidade, redes WDM e redes fotônicas.

Keywords: Interconnection network, network topology, performance analysis.

1. INTRODUCTION

With the evolution of the switching networks and multiprocessing systems, the interconnection networks play an important role in the improvement of the performance of communication and computer systems. Connecting network nodes with the shorter possible delays is one of main goals of any interconnection network, without saying that using efficient interconnection networks is critical to the performance of large communication networks with hundred or thousand of communication elements (nodes). In addition, it is advisable for each particular application to optimize the performance parameters, such as the packet loss rate and throughput of the interconnection network, by implementing suitable routing strategies and conflict resolution rules.

The multi-dimensional toroidal networks have some particular performance characteristics that make them very suitable for many applications [1, 2]. This paper introduces a new multi-dimensional network architecture arranged in alternated regular mesh fashions with toroidal boundaries. The new network topology preserves the main characteristics of many toroidal networks such as: isotropy, easy routing and fast node identification. Some applications of toroidal networks include the Manhattan Street Network developed by N.F. Maxemchuk [3], the HR4-Net proposed by Borgonovo and Cadorin [4], a toroidal based PAX series multiprocessor from the Institute of Engineering Mechanics at the University of Tsukuba [5], and parallel computers (ILLIAC IV, Massively Parallel Processors, Distributed Array Processors and Wire Routing Machine) [6].

Attributes of an interconnection network include: the *diameter, bisection width, symmetry, deflection index* and *connectivity degree*. It is desirable that these attributes (or characteristics) of an interconnection network can be determined by the network topology and are possibly independent of the incurred network traffic characteristics, the input traffic volume, the employed routing mechanism, and the applied congestion control strategy. However, only those interconnection networks with a static topology, where nodes are permanent, are able to maintain their attributes reasonably the same while those assuming a dynamic topology frequently have their attributes easily changed whenever the topologies are altered from time to time. In this introductory section, we provide some definitions to these network attributes and introduce the concept of network capacity via Little's Theorem.

The *diameter* is the maximum of the shortest distance (hops) between any two nodes. A precise definition of the diameter for interconnection networks could be as the following: *"the maximum of the shortest distance between any two nodes in the network"*. Mathematically, the diameter (δ) of an interconnection network can be expressed by

$$\delta = \max_{1 \leq 1, j \leq N} \{m_c(I, J)\}, \tag{1}$$

where N denotes the number of nodes in the network and $m_c(I, J)$ represents the distance measure of the shortest path between nodes I and J. For *multi-hop* networks, the diameter is an attribute highly representative and related to the maximum information transfer delay. In cases where a deflection routing strategy is adopted, the diameter represents the maximum transfer delay of the packets in the network without deflection. Therefore, intuitively the difference between the network diameter and the maximum transfer delay would become small if the link occupation rate is low due to low packet deflection probabilities.

The *bisection width* of an interconnection network can be defined as the following [7]: *the minimum number of links that have to be removed to disconnect the network into two halves with an identical number of nodes (or within one node of difference).* In fact, bisection width is a critical factor in determining the performance of a network because in most scientific problems, the data contained and/or computed by one half of the network are needed by the other half. Therefore, it is recommended to use networks with large bisection width so that high efficiency in communication between two halves can be achieved. In addition, large bisection width promotes a higher degree of system's fault tolerance. It is also worth mentioning, however, for VLSI circuits, the larger is the bisection width, the higher will be the circuit implementation cost. Hence it would be necessary to ponder the advantages and disadvantages of a large bisection width according to applications considered. Among the systems partially or totally VLSI implemented we cite: multi-computers or multi-processsors and broadband network switches.

The *deflection index* of a network is defined as the least upper bound over the number of hops a single deflection adds to the packet's delay [8]. It is easy to sense that the deflection index depends on both network topologies and routing algorithms. Like diameter, the deflection index is directly related to the network transfer delay. An ideal interconnection network in terms of small delay should reduce the deflection index as well as its diameter. Optimization of these two attributes in designing new network topologies could be a considerable challenge. For some topologies such as mesh and toroidal networks, the deflection index can be defined without referring to any particular routing algorithm. In this case the deflection index may be alternatively defined as: *the length of a shortest round-trip path for a given network topology* [9].

The *degree of connectivity* of an interconnection network can be defined in two manners, with respect to the node's incoming and outgoing links. The input (output) degree of connectivity, θ_{in} (θ_{out}), represents *the number of incoming (outgoing) links connected to a given network node.* Interconnection networks that have the same input and output degree of connectivity for all network nodes are classified as *regular topology networks*. A regular network is "p-connected" when its degree of connectivity is p.

Different applications impose different limits on the degree of connectivity of interconnection networks. Borgonovo's argument registered in [10] claims that for local and metropolitan networks, indiscriminate increment of the degree of connectivity results in high costs; therefore, the utilization of an efficient routing algorithm is preferable and in general network topologies with the degree of connectivity larger than 4 are not considered. For the case of optical networks and broadband switching in which routing time becomes a limiting factor in network design, the degree of connectivity should be high enough to accommodate high network transmission rates. For parallel computing systems, most proposals have adopted a degree of connectivity not larger than 6 [11, 12, 13, 14].

The definition of *symmetry* of interconnection network topologies encompasses the concept of the isomorphism and automorphism established in the graph theories. Two graphs (or topologies) **G** and **H** are said isomorphic if there is an one-to-one correspondence between the links of **G** and **H** [15], i.e., if **H** can be obtained from renaming links in **G**, and vice versa. The automorphism of graph **G** represents the isomorphism of **G** with respect to the proper graph **G** [16]. A network is symmetric, if for any pair of nodes "a" and "b", there is an automorphism of the graph that maps "a" to "b". In other words, the network just "look" the same from any node in terms of topological homogeneity. Such a property is highly desirable for practical implementation of interconnection networks because the homogeneity of nodes allows the use of the same local routing algorithm. It is worth mentioning that many advantages of using a local routing algorithm over a centralized routing algorithm include: higher fault tolerance, more flexibility in system routing management and network scalability.

In general, symmetric networks allow that stochastic analysis being carried out, which possibly results in the formulation of some probabilistic models. From these models, many network performance measures, such as mean throughput and mean transfer delay, can be obtained analytically, avoiding exhaustively time-consuming simulation tasks.

Performance parameters of an interconnection network are considered as dynamic variables that depend not only on network topologies but also on traffic patterns, traffic intensities, and applied routing algorithms. These variables are capable of providing essential information in deciding the best use of the network in practice. The principal performance parameters used to evaluate a network include: throughput, transfer delay, channel utilization and network capacity. Next, we define precisely these parameters in the context of interconnection networks (INs). Let

- $P(t)$ = number of packets in the IN at time t;
- N_t = number of packets accepted by the IN during the interval $[0, t]$;
- T_i = time spent in the IN by the i^{th} arriving packet.

The time average throughput (λ_t) of an interconnection network for interval $[0, t]$ can be defined as

$$\lambda_t = \frac{N_t}{t}. \quad (2)$$

Note that λ_t equals to the average arrival rate if the considered IN is lossless. As t increases, the throughput λ_t tends to the steady state time average throughput value λ, which can be always achieved by a positive recurrent communication network [17]. In network simulation, the steady state is declared when the time average packet acceptance rate is equal to the time average packet departure rate. Mathematically, the steady state throughput of an interconnection network is defined as

$$\lambda = \lim_{t \to \infty} \lambda_t. \quad (3)$$

Frequently, a normalized version of the network throughput, with respect to the node transmission rate, is used. Therefore, for networks with multi-link connections the normalized throughput in general assumes a value larger than a unit. The network throughput is an important performance parameter capable of deciding whether the network supports or not a certain traffic volume required by the application. In other words, it defines the potentiality and practicability of the interconnection network considered.

The time average network information transfer delay (T_t) for jobs which arrive in the interval $[0, t]$ can be defined as

$$T_t = \frac{\sum_{i=1}^{N_t} T_i}{N_t} \quad (4)$$

where T_i includes the total time spent by the i^{th} accepted packet in the IN system. The steady-state network information transfer delay is defined as

$$T = \lim_{t \to \infty} T_t. \quad (5)$$

Like throughput, the network information transfer delay also sets some limitation on the utilization of the network in practice. For instance, off-line transfer of voice and images frequently is not tolerable [18, 19].

Two elements are needed to define *channel utilization*; they are the time-average of number P of packets and number l of links in the network. Let $P(\tau)$ denotes the traffic intensity. If the "typical" number P_t of packets in the network observed up to time t is given by

$$P_t = \frac{1}{t} \int_0^t P(\tau) d\tau, \quad (6)$$

P_t tends to P as t increases, i.e.,

$$P = \lim_{t \to \infty} P_t. \quad (7)$$

The number l of links in the network is a static variable which depends exclusively on the network topology, and can be defined as the total number of end-to-end links comprising a topology of the network. Hence, the steady state channel utilization U in an interconnection network can be defined as "*the fraction of average time that the network links remain busy (information transmission) when the network operates in the steady state*" [20]. Under the assumption that each network link serves up to one packet transmission per unit time slot, the channel utilization is given by

$$U = \frac{P}{l} \quad (8)$$

In this case, since the number of links in a network represents the maximum number of packets that the network is able to accommodate for a given time point, the network channel utilization reflects a measure of network efficiency with respect to the maximum transmission rate of the network.

Three of the above defined parameters, throughput, transfer delay and channel utilization, can be related to each other by Little's result:

$$\lambda = \frac{P}{T} \quad (9)$$

In terms of a queueing system, the formula concludes that the average number of customers in the system is equal to the product between the average customer arrival rate and the average time that each customer spends in system queue.

All network performance parameters described so far depend largely on the network traffic volume as well as the applied routing strategies. It would be highly illustrative to be able to predict the utmost performance that a new designed network system can achieve. Interconnection Network *Capacity*, which is expected to be independent of the network traffic volume, and a candidate for this end, represents *the maximum achievable steady state throughput* [21]. Network capacity (C) is a function of network topologies and routing strategies, mathematically expressed as

$$C = \max_{r \in R} \lambda \quad (10)$$

where R is the set of all applicable routing algorithms (r).

This paper is organized as follows: in Section 2 we define the D-ARM topology and briefly describe three conflict resolutions rules: **random**, **straight-through** and **closest to finish**. Section 3 shows explicitly the formulas for determining the attributes of the D-ARM topological networks. The main issue of Section 4 is the derivation and comparison of upper bounds for the information transfer capacity of the D-ARM networks with different dimensions and numbers of nodes. In Section 5 we compare the performance of the D-ARM networks to other two well-known interconnection networks: ShuffleNet and MSN. Finally, in Section 7 we conclude with some possible applications of the D-ARM networks and suggestions for future investigation.

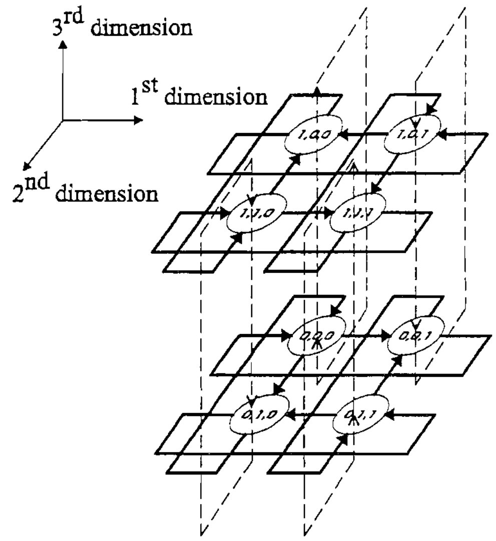

Figure 1. Example of a 3-ARM Network - 2x2x2.

2. THE D-ARM NETWORK TOPOLOGY

The new proposed network architecture has a multi-dimensional topology arranged in alternated regular mesh fashion (D-ARM). The borders of the new network are connected in a toroidal way to avoid the borders effects [1], and therefore to reduce the distance between nodes. Each node of a D-ARM network has D incoming links and D outgoing links. The node address inside the network is represented by a D-dimensional vector $I = (i_D, ..., i_2, i_1)$ where each entry of the vector I is a non-negative integer value. Figure 1 shows an example of 3-ARM with 8 nodes.

In order to facilitate the D-ARM network representation and analysis we associate the orientation of network links to the orientation of coordinate axes of a D dimensional vector space, i.e., 1^{st} dimension links, 2^{nd} dimension links, ... , or D^{th} dimension links, as shown in Figure 1. Note that in a D-ARM network each network node has just one incoming link and one outgoing link at each one of D dimensions (or directions). The direction of an outgoing link can be for (increasing) or against (decreasing) with respect to the orientation of the corresponding coordinate axis. To find the direction of a j^{th} dimension outgoing link of node I, we apply the following rule:

$$\text{If} \left(\sum_{n=1}^{D} i_n \right) - i_j = \begin{cases} \text{even,} & \text{the link is decreasing} \\ \text{odd,} & \text{the link is increasing} \end{cases} \quad (11)$$

Another way to look at a D-ARM network is to consider a set of rings, each of which is formed by a group of nodes connected by links all having the same orientation. The length of each ring corresponds to the number of nodes in the ring. Let l_1, l_2, ..., and l_D denote the length of the rings at the 1^{st}, 2^{nd},..., and D^{th} coordinate dimensions, respectively. In this work, we consider only the case of $l_1 = l_2 = ... = l_D = W$ which is a necessary condition to have networks presenting the symmetric, regular and toroidal properties simultaneously. Hence the total number (N) of nodes in a D-ARM network of this sort with length W is WD and the k^{th} entry (i_k) of node $I = (i_D, ..., i_2, i_1)$ is an integer number varying from 0 to W-1. Interesting enough is that the topology of a D-ARM network now is completely defined by its dimension D and length W. Moreover, in order to preserve the network's toroidal boundary connection pattern, the length W must be an even number.

Due to the global isotropy propriety of the D-ARM networks [2], a distributed and self-routing algorithm that identifies shortest paths, based only on the addresses of the source and destination nodes, can be easily developed and implemented. It may happen that at a given node, an outgoing link is disputed by two or more packets. Under such circumstances, a *contention resolution rule* should be invoked to solve the conflict. Some basic contention resolution rules, frequently adopted by interconnection networks, include the following deflection strategies:

- **random**: the conflict is resolved by a random choice among the conflicting packets;
- **straight-through**: the packet is sent via the outgoing link in the same direction (dimension) as the incoming link;
- **closest to finish**: the preference is given to the packet near to its destination. If two or more packets are equally away from their destinations, the conflict is resolved by a random choice.

The D-ARM network is a slotted packet communication system where each node can receive up to D packets from its incoming links and generate a new packet per time slot $(t-1, t)$. In the following time slot $(t, t+1)$ each node tries to send all packets (received + generated) through their D outgoing links applying a routing algorithm. By assumption, the packets already found in the network have higher priority than a new packet in disputing an outgoing link. As a consequence, a new packet can be sent if at most $D-1$ routing packets are received or if at least one routing packet is addressed to the node.

3. TOPOLOGICAL ATTRIBUTES OF THE D-ARM NETWORKS

By either simulation or analysis and considering all shortest paths, we determined the diameter (δ) of a D-ARM network with length W as follows:

- If W is odd, $\delta = \frac{D(W-1)+2}{2}$
- If W is even and 4 does not divide W, $\delta = \frac{DW}{2}$
- If W is even and 4 does divide W, $\delta = \frac{DW+2}{2}$

Note that when a D-ARM is symmetric, which is the second case above, the network diameter (δ) can be easily derived from simple reasoning. When the network does not have the symmetric properties (the first and third cases above), we can find the network diameter by running a Flooding algorithm [17].

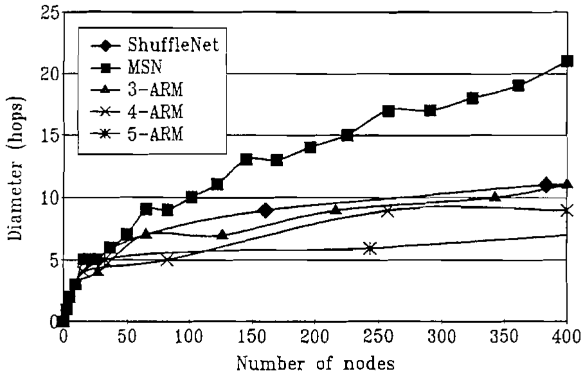

Figure 2. The diameter for different network topologies.

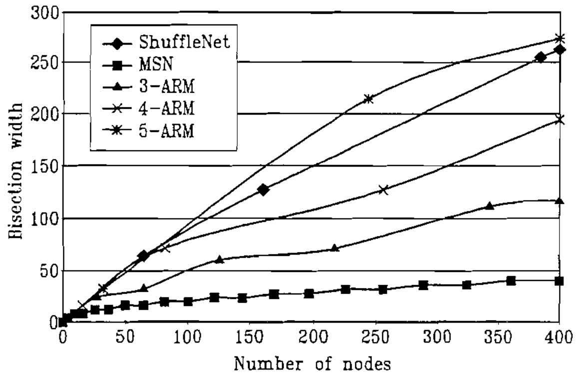

Figure 3. The bisection width for different network topologies.

Figure 2 shows how the diameter varies with the number of nodes for topologies of ShuffleNet, MSN and D-ARM networks up to 400 nodes. Note that, for small networks ($<$ 50 nodes), no remarkable difference in terms of the diameter value is observed for these network topologies. However, for a large number of nodes, the SuffleNet and D-ARM topologies are able to maintain the network diameter considerably low in comparison with the MSN topology. Since the ShuffleNet topology belongs to the group of topologies of minimum diameter [8], we adopt it as a reference merit figure to evaluate the diameter of the D-ARM topology networks. For networks up to 400 nodes, a 3-ARM network is enough to keep the network diameter smaller than or equal to that of the ShuffleNet. Even for networks with a very large number of nodes, there is still no need to increase the dimension of the D-ARM topology beyond D =4 or 5 in order to keep the network diameter as low as that of the ShuffleNet. For instance, in designing a network with 10,240 nodes, the diameter of the ShuffleNet is of 19 jumps (hops) while 20 and 16 jumps are found to be the diameter for the 4-ARM and 5-ARM, respectively.

For the D-ARM topology, the *bisection width* (β) is a function of the network dimension (D) and length (W):

- If W is odd, $\beta = 2W^{D-1} + 2W^{D-2}$
- If W is even $\beta = 2W^{D-1}$

Figure 3 compares the bisection width of the ShuffleNet, MSN and D-ARM topologies up to 400 nodes. The ShuffleNet topology presents bisection widths considerably superior to the ones of MSN topology. For the D-ARM topology, the larger is the number of nodes, the higher should be the dimension degree of the D-ARM so that the D-ARM networks outperform the SuffleNet with regard to bisection width. For example, for networks with an amount of nodes varying between 100 and 400, a D-ARM network with D = 5 or higher should be employed. As mentioned before, large bisection width is essential for low information transfer delay in networks with traffic uniformly distributed, because, in this case, there is a high traffic flow of information between the two halves of the supposedly divided subnetworks. On the other hand, as in most of practical applications involving large networks ($>$ 1000 nodes), when the information routing involves some small parts of the networks, the variation in bisection width in general has little effect on transfer delays.

The *deflection index* (ϕ) is another network parameter related directly to the network transfer delay. Since both MSN and D-ARM topologies are connected in toroidal fashion, their deflection indices are constant, i.e., $\phi = 4$, independent of both the network dimension (D) and length (W). In contrast, the *deflection index* in the ShuffleNet topology is proportional to the number of nodes. Clearly the ShuffleNet topology should be avoided in a network design if a low deflection index is required. Note that a deflection index of 4 in the MSN and D-ARM topologies is only achieved by a ShuffleNet with 64 nodes.

For high speed interconnection networks, the degree of connectivity imposes a limitation on the utilization of high dimensional D-ARM topologies. This is due to the fact that the D-ARM input and output *degrees of connectivity* are exactly equal to the network dimension value (D), i.e.,

$$\theta_{in} = \theta_{out} = D \tag{12}$$

In fact, a high degree of connectivity means more complexities involved in making routing decisions and in real network implementation. In order to have networks working properly and being stable, the time spent in packet routing should not be larger than the mean packet interarrival time. A simple computation reveals that the routing complexities, in terms of number of possible routing choices, increase exponentially with the total number of outgoing links at a given node. For example, as shown in Table 1, in a p-connected topology, there are $p!$ and p^p different ways to direct p packets to their p outgoing links using the deflection routing and the store-and-forward routing strategies, respectively. Table 1 and Table 2 provide a rough estimate of the minimum number of clock cycles required for an ATM network with 1 Gbps data rate and 10 GHz clock frequency. Based on this information we conclude that the dimension value (D) of the D-ARM network should not exceed 5 and 4 when the *deflection* routing and the *store-and-forward* routing algorithms, respectively, are used.

4. THE D-ARM NETWORK PERFORMANCE

To evaluate the performance of the D-ARM network topologies, we tested two well-known conflict resolution rules (**random** and **straight-through**) [8] and introduced

	Deflection Routing	
D	Routing Options	Clock Cycles
2	2	4
3	6	18
4	24	96
5	120	600
6	720	4.320
7	5.040	35.289
8	40.320	322,560
16	$2.09x10^{13}$	$3.344x10^{14}$

Table 1. Routing complexities in interconnection D-ARM networks using the deflection routing.

	Store-and-forward Routing	
D	Routing Options	Clock Cycles
2	4	8
3	27	81
4	256	1,024
5	3,125	15,625
6	46,656	279,936
7	823,543	5,764,801
8	16,777,216	134,217,728
16	$1.84x10^{19}$	$2.9x10^{20}$

Table 2. Routing complexities in interconnection D-ARM networks using the store-and forward routing.

a new conflict resolution rule, the so-called **preferential** rule. The preferential conflict resolution rule assigns an outgoing link to a routing packet according to its degree of preference (G_p), which is defined as the total number of optimum paths available at the moment of conflict. Note that the degree of preference ranges from 0 to D. Once the degrees of preference of all routing packets are determined, the preferential conflict resolution rule assigns outgoing links to the packets based on the following criteria:

1. The packets with the lowest G_p have the highest priorities to choose their preferable outgoing links;
2. $G_p = 0$ is attributed to the packets that have lost the possibility of choosing an optimum path;
3. A random strategy will be invoked to assign unused outgoing links to the packets with $Gp = 0$;
4. The same random strategy will be invoked whenever the number of packets with the same G_p is larger than the value of G_p.

Figures 4 and 5 respectively show how the throughput and delay of the 3-ARM network (64 nodes) change with the new packet generation rate (g) for three conflict resolution strategies (random, straight-through, preferential) described before. It is assumed that per each time slot each node has the probability g of generating a new packet and the packets' final destination nodes are uniformly distributed. Note that no remarkable difference in throughput (1.15%) and delay(1.42%) is observed among the three deflection

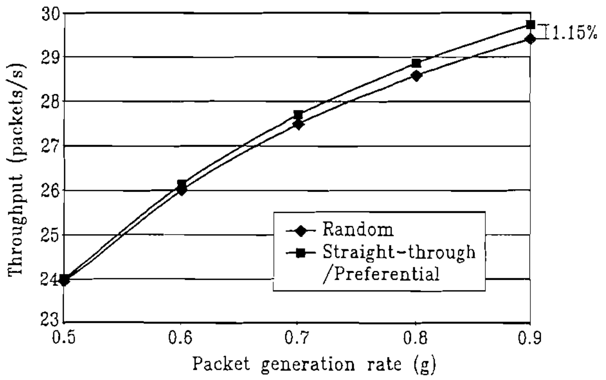

Figure 4. The throughput of the 3-ARM (64 nodes) network under the random, straight-through and preferential conflict resolution rules.

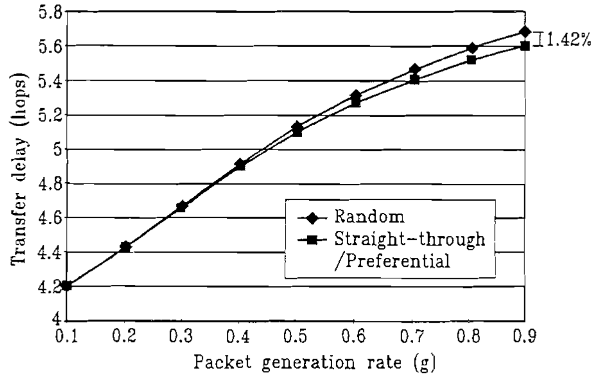

Figure 5. The delay of the 3-ARM (64 nodes) network under the random, straight-through and preferential conflict resolution rules.

strategies. In addition, the preferential and straight-through techniques present the same performance in terms of throughput and delay.

Repeating the same simulation procedure, however, on the 4-ARM network (256 nodes), we obtained the network performance (Figures 6 and 7) similar to that of 3-ARM (Figures 4 and 5). Note that the preferential and straight-through techniques no more result in the same performance measures as did in the case of the 3-ARM network although the differences in these performance measures are very small. Moreover, the differences in throughput and delay between the random and preferential techniques have increased from 1.15% to 2.00% and 1.4% to 2.44%, respectively, when our investigation advanced from the 3-ARM network to the 4-ARM network. In spite of the increase, the differences in throughput and transfer delays are still considerably small between two conflict resolution rules.

4.1. BOUNDS FOR TRANSMISSION CAPACITY

As mentioned before, network transmission capacity represents the network's maximum achievable throughput. Precise evaluation of the network capacity is by no mean a trivial task because we do not have knowledge about all factors that affect routing. In this work, we concentrate our effort on an attempt to find an upper bound to the D-ARM

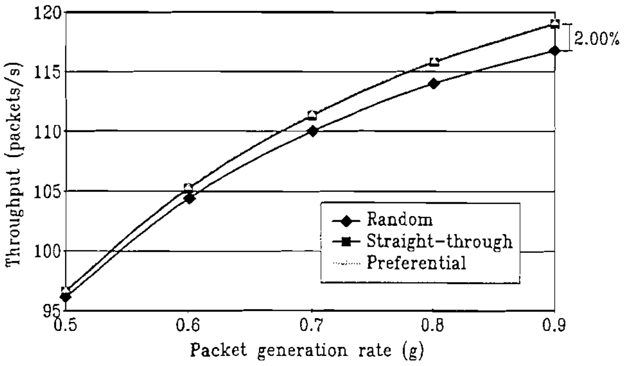

Figure 6. The throughput of the 4-ARM (256 nodes) network under the random, straight-through and preferential conflict resolution rules.

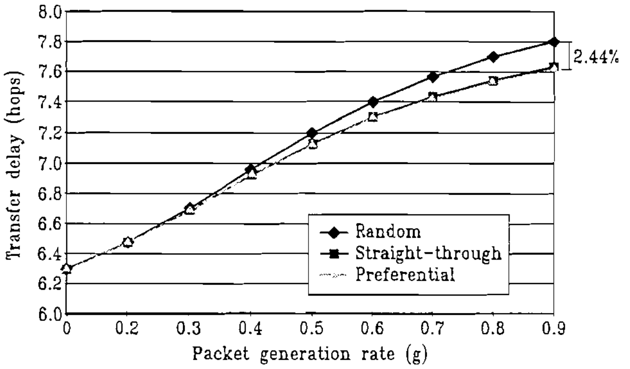

Figure 7. The delay of the 4-ARM (256 nodes) network under the random, straight-through and preferential conflict resolution rules.

network capacity, and then compare this upper bound to that of networks with different topologies. Most important, this upper bound of the network capacity provides us with some indication about what would be the utmost throughput.

The mean number ($\bar{N}$) of packets in a D-ARM network is upper bounded by the total number of outgoing links as:

$$\bar{N} \leq DW^D \tag{13}$$

The transfer delay (T) between the source node I_s and the destination node I_d can be written as:

$$T \leq sp(I_s, I_d) + 4F \tag{14}$$

where $sp(I_s, I_d)$ denotes the length of the chosen shortest path from I_s to I_d, and F denotes the number of deflections from shortest paths. On D-ARM networks when a packet is deflected, the length of its path can be increased by 4 hops. In other words, the deflection index of the D-ARM networks is just 4. On the other hand, the mean transfer delay ($\bar{T}$) is lower bounded by the mean length of the shortest path, i. e.,

$$\bar{T} = E\{T\} \geq E\{sp(I_s, I_d)\} \approx D\frac{W}{4} = \frac{DW}{4} \tag{15}$$

Note that we approximate the mean length of the shortest paths ($E\{sp(Is,, Id)\}$) by $\frac{DW}{4}$ based on the fact that the diameter on each ring of a toroidal network is $W/2$ [1],

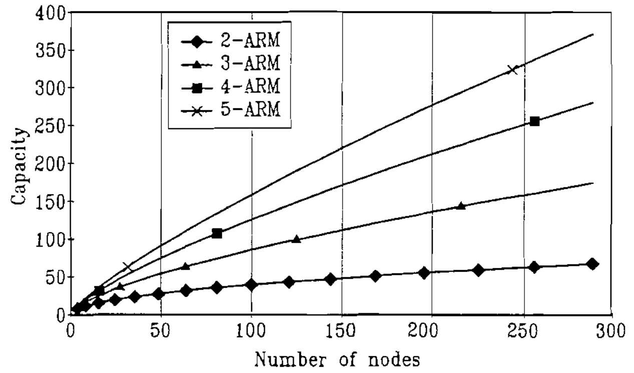

Figure 8. The upper bound of the D-ARM network capacity versus number of nodes.

and therefore the mean distance between two nodes of the ring is $W/4$. We have verified, via computer simulation, that the suggested delay value, although not exact, is a good approximation for the D-ARM network with error less than 1% with respect to the real mean distance between two nodes. In addition, the larger is the network, the smaller errors will be.

The steady state throughput ($\bar{\lambda}$) of a D-ARM network can be obtained by Little's Theorem and is upper bounded by

$$\bar{\lambda} = \frac{\bar{N}}{\bar{T}} \leq 4W^{D-1}. \tag{16}$$

Since no routing algorithm is explicitly mentioned, the upper bound for $\bar{\lambda}$ suggested in Equation (16) should be valid for all applicable routing algorithms including the optimal ones. Considering the set R of all applicable routing algorithms, the D-ARM network capacity is upper bounded by

$$C = \max_{r \in R}\{\bar{\lambda}\} \leq 4W^{D-1} \tag{17}$$

The use of networks with higher capacity reduces packet loss rates and deflection probabilities. High dimension networks are suitable for applications where small transfer delays and low cell loss rates are expected; e.g., ATM switching. Figure 8 shows the upper bound of the D-ARM network capacity versus number of nodes. From the plot we conclude that the larger is the network dimension, the higher is the network capacity. Such a conclusion is intuitively plausible due to the following observations: (a) a higher order D-ARM network is able to accommodate more packets and reduce the probability of the network becoming congested; (b) for a fixed number of nodes, a higher dimensional D-ARM network presents a smaller diameter; therefore, a smaller packet transfer delay.

We also made a brief analysis about the cost which would be incurred in implementing the D-ARM networks. The cost parameter would provide us with some indications about the most adequate network dimension and length to be chosen for each application.

Reed and Grunwald [13] have defined a cost function that permits the analysis of different networks topologies:

$$Cost = C_{node}N + C_{link}DN + C_{con}2DN \tag{18}$$

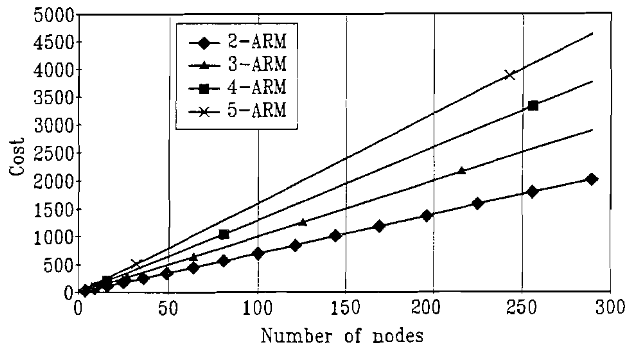

Figure 9. The D-ARM network cost versus number of nodes.

Topology	Number of Nodes (N)
ShuffleNet	2, 8, 24, **64**, 160, 384 ($N = k2^k$)
MSN	4, 9, 16, 25, 36, 49, **64**, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400 ($N = k^2$)
3-ARM	8, 27, **64**, 125, 216, 343 ($N = k^3$)
4-ARM	16, 81, 256 ($N = k^4$)
5-ARM	32, 243 ($N = k^5$)

Table 3. Feasible numbers of nodes for different network topologies.

where N, D, C_{node}, C_{link} and C_{con} are the number of network nodes, network dimension, node cost, link cost and connection cost, respectively. Adapting to the suggested cost function to the case of the D-ARM networks where $N = W^D$, via Figure 9 we show how the cost function of the D-ARM networks varies in terms of the number of nodes with the node cost, link cost and connection cost all assuming the unit value. Under the linear assumption, with a fixed number of nodes, the cost varies linearly with the network dimension (D). It is worth remembering that we should not increase the network dimension size (D) unless it represents a significant decrease in the diameter value as well as the network cost.

5. PERFORMANCE COMPARISON

In this section we compare the performance measures considering 3 network topologies: MSN, ShuffleNet and D-ARM. The analysis of the network performance is based on throughput, delay and upper bounds of network capacity defined in the previous section. Since these network topologies may not be defined for any arbitrary number of nodes, it seems imperative to establish a common base in order to reach a meaningful comparison. For the network throughput and delay comparison, we demand that the number of nodes in networks be the same and the same routing strategy be applied. Simple computation reveals that in the universe of 2 to 400 nodes and network dimension (D) no greater than 5, the networks of 64 nodes are the only feasible case as shown in Table 3. In other words, we perform our comparison analysis on the 3-ARM (4x4x4), the 64-node ShuffleNet and 8x8-node MSN.

Figure 10 plots the throughput of the three networks in

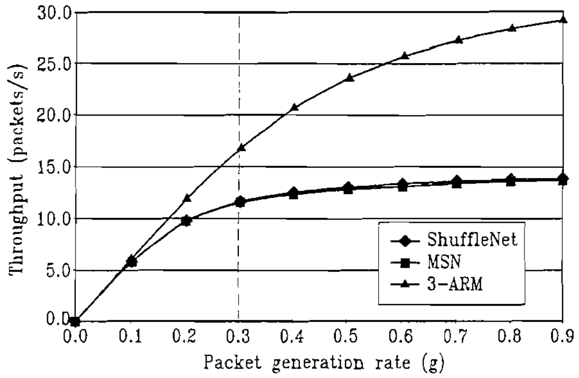

Figure 10. Throughput versus packet generation rate for the 3-ARM(4x4x4), the ShuffleNet-64 and the MSN-8x8.

function of new packet generation rate (g) in each node under the deflection routing and random conflict resolution rule. Our analysis on the behavior of these three throughput curves is based on the concept of **saturation rate**, defined as the lower bound of an interval in which network throughput gradient (or growing rate) is equal to or less than 10% of the gradient of the throughput in an interval of low values of g, e.g,. $[0, 0.3]$. Such a definition of the gradient, instead of adopting strictly the generating rate at which occurs the 90% of the saturated throughput value, is frequently convenient due to the fact that some throughput curves may not present "saturated" behavior. For the ShuffleNet and MSN, we found that the throughput gradient is $\cong$ 39.3 in the interval $[0, 0.3]$ of generating rate g. Taking this interval ($[0, 0.3]$) of generating rate g as a reference for comparison, we found the throughput gradient falling below 3.93 occurring in the interval $[0.5, 0.6]$ of generating rate g. Hence, in this case the saturation rate is taken as $g_s = 0.5$. On the other hand, for the 3-ARM network, the throughput gradient in the interval $[0, 0.3]$ of the generating rate is approximately 56.7 and therefore $g_s = 0.9$. Comparing these derived saturation rates, we found evidently that the 3-ARM network is much superior to the other two (ShuffleNet and the MSN) in terms of the capacity of supporting high traffic volumes.

Figure 11 compares the amount of transfer delay introduced by the 3-ARM, the MSN and the ShuffleNet operating at different generating rates. The transfer delay in the ShuffleNet and MSN is at least 59% larger than that in the 3-ARM network. In addition, the 3-ARM network presents a smaller variation in transfer delay with the variation of the generation rate g. Such a feature brings considerable advantage in applications of high speed networks in which the system response should not vary considerably during some sporadic fluctuations of traffic volumes.

A comparative study of upper-bounds of transmission capacity of different networks allows us to infer the potential of information transfer of these networks, and help us to choose the most adequate one in practical applications. Figure 12 plots capacity upper-bound curves of the ShuffleNet [9, 10], MSN, 3-ARM, 4-ARM and 5-ARM. Analyzing these curves, loosely speaking, a 5-ARM network with 400 nodes has potential of offering a traffic volume about 243% and 381% more than the SuffleNet and the MSN,

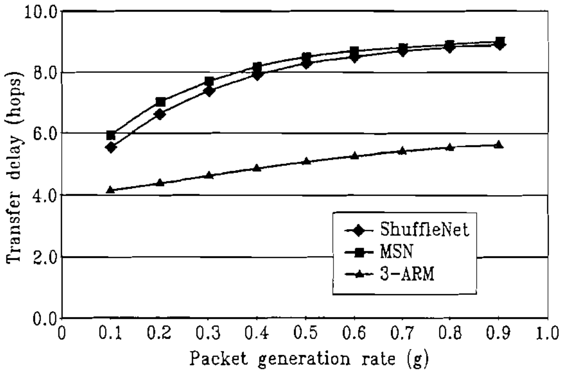

Figure 11. Delay versus packet generation rate for the 3-ARM(4x4x4), the SuffleNet-64 and the MSN-8x8.

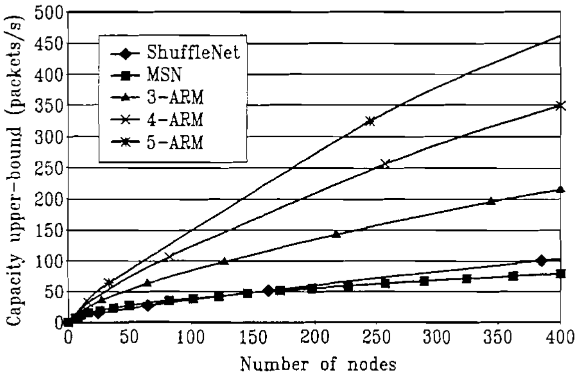

Figure 12. Capacity upper-bound curves of the ShuffleNet, MSN, 3-ARM, 4-ARM and 5-ARM.

respectively.

6. COMPUTER SIMULATION

The performance evaluation of the D-ARM was done via computational simulation. For this end several simulation modules were implemented in C++ and over UNIX (Sparc 1000) and PC platforms. Here we describe some major elements of the proposed D-ARM simulation model.

A discrete time simulation and event oriented model was chosen for the *D*-ARM modeling. In other words, the system is modeled by defining the state change for each event. Therefore, the system state remains unchanged in a period between two consecutive events. Each event can be classified into a primary or a secondary event, and each primary event is scheduled directly by the simulator according to the timing information. A secondary event, which is not previous scheduled, depends on the primary event that causes its appearance [26].

The following stop criterion was used in the simulation:

- The absolute value of the difference between the packet output rate at time t and the packet output rate at time t+1 is less than 10^{-3};
- The absolute value of the difference between the packet input rate at time t and the packet input rate at time t+1 is less than 10^{-3};
- The absolute value of the difference between the packet output rate at time t+1 and the packet input rate at time t+1 is less than 10^{-3}.

The value of 10^{-3} was chosen experimentally among many values ($10^{-3}, 10^{-4}, 10^{-5}$ and 10^{-6}) because it establishes a good equilibrium between the precision in simulation results and the duration of the simulation. Under the chosen stop criterion (10^{-3}) we have observed the differences between the performance parameters obtained from consecutive simulations is less than 1.0% whenever the number of samples (input packets) is larger than 50000.

With respect to the traffic pattern used in the simulation, we adopted the uniform distribution and assumed that the traffic generation is done independently among nodes but using the same rate. Each packet's destination is randomly and uniformly chosen among the remaining N-1 nodes.

In order to validate our simulation model and provide a high degree of confidence about the simulation results, the following procedure was performed:

- The calculation of each kind of performance measure (throughput, delay and channel utility) is done independently;
- Using Little's Theorem to verify the consistency of the values obtained from simulation.

In other words, all the values of througput, transfer delay and channel utilization obtained from the simulation must satisfy Little's Theorem with error never superior to 0,1%.

7. CONCLUSION

This paper proposes a new multi-dimensional network architecture (*D*-ARM) where each node has *D* outgoing links arranged in alternated directions. The analysis of its topological attributes and performance parameters suggests some practical applications which include: Computer Networks, WDM Optical Networks, Processors Interconnection and Broadband Switching Architectures.

Maxemchuck [3], suggesting the MSN network (2-ARM), reinforces the advantages of the mesh topology over the conventional topologies such as ring and bus topologies. The replacement of networks with conventional topologies by mesh ones may increase the transmission throughput without necessity of augmenting the packet generation rate. This is due to the mesh network's high connectivity and the nodes' parallel processing that result in more packets accepted by the network. In addition, the reliability of the mesh networks is considerably higher than conventional MAN's and LAN's without mentioning the possibility of being used on wide area networks.

In the optical connectivity layer, defined by Acampora [22], it is necessary to have a streamlined connection network with high transmission capacity and short transfer delay. Some interconnection networks, such as ShuffleNet, have been already considered for this application. In regard with the analysis results developed in Section III, the *D*-ARM networks can be successfully adapted to large optical networks with high capacity and extremely short delay.

Reed and Grunwald [11] have analyzed the performance of many interconnection network topologies. Among the standing out topologies are the N-cube and D-torus. They analyzed only a 3 dimensional D-torus (similar to the 3-ARM) and showed that for a network up to 1000 nodes the topology the D-torus presented the highest routing packet rate. Further investigation is necessary to establish the advantages and disadvantages of the D-ARM networks over the other processor interconnection network topologies.

Since we impose that D-ARM model present a symmetry topology, the length of all D-ARM networks proposed in the paper is even. Such a requirement no doubt will set some limitation to the practical applications of the model.

Concerning broadband switching architecture there are two fundamental points that must be emphasized: the transfer delay and the cell loss probability. We conclude that the D-ARM network can perform better than some well known architectures such as tandem banyan switching fabric (TBSF) [22] and Shuffle network [23], specially for $D = 3, 4$ or 5. Nevertheless for a larger D, ShuffleNet outperforms the D-ARM model due to its larger bisection width. In addition, fast packet or information switching frequently requires the use of simple contention resolution and routing strategies, therefore, an IN module with a low degree of connectivity. Such a restriction may again limit the use of high order D-ARM networks.

We have no doubt that a more detailed investigation is needed in order to better evaluate other performance parameters of the D-ARM networks. Parameters such as packet loss probability, deflection probability and mean transfer delay are important to determine accurately all the advantages and disadvantages of the D-ARM networks over the other interconnection networks. It is also desirable to develop a stochastic model which best describes the dynamic proprieties and accurately evaluates all performance parameters of the D-ARM networks. It is worth mentioning that stochastic models such as, one node model [24] and signal flow graphs [25] can be generalized to the case of the D-ARM networks. In addition, further improvement on the performance of the D-ARM networks can be done, such as including input queue in order to lower effective packet loss rates, possibly achieving a zero packet loss rate.

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