EFFICIENT FINITE ELEMENT MODELLING OF OPTICAL WAVEGUIDES WITH ARBITRARY CURVED INTERFACES

H. E. Hernández-Figueroa, *Senior Member IEEE*

And

M. Silveira, *Member IEEE*
Universidade Estadual de Campinas (UNICAMP)
Faculdade de Engenharia Elétrica e Computação (FEEC)
Departamento de Microonda e Óptica (DMO)
UNICAMP-FEEC-DMO
Caixa Postal 6101 13083-970 Campinas - SP, Brasil
Tel. : +55-19-2397415 Fax : +55-19-2391595
E-Mails: hugo@dmo.fee.unicamp.br and
silveira@dmo.fee.unicamp.br

RESUMO - Este artigo traz uma abordagem numérica eficiente na análise de guias ópticos com interfaces dielétricas constituídas por curvas suaves. A estratégia aqui adotada aglutina três aspectos numéricos: um reticulado eficiente, uma forma personalizada eficiente de resolver problemas de autovalores generalizados com o uso de subrotinas de convergência rápida; e a utilização apropriada das condições de contorno. O algoritmo implementado para o reticulado adaptativo da secção transversal do guia óptico faz uso de elementos finitos triangulares lineares, e de primeira ordem (lados retilíneos), os quais produzem um alto grau de precisão, sem ter que recorrer aos elementos finitos curvilíneos, e de segunda ordem. A comprovação deste tipo de abordagem numérica está sendo qui apresentada com o cálculo dos modos e das respectivas curvas de birefringência em guias com núcleos elípticos que são referenciados na atual literatura. Destacamos também a análise de um acoplador simétrico 2x2 à fibra fusionada, colocando em evidência alguns parâmetros críticos que comparecem no projeto deste tipo de estrutura óptica.

Abstract — In this paper an efficient numerical approach based on the finite element method to analyse optical waveguides with smooth and arbitrary shaped dielectric interfaces is reported. Our strategy combines three numerical aspects: an efficient refinement, an efficient sparse matrix solvers, and proper boundary conditions. The adaptive mesh refinement algorithm takes into account simple first order linear (straight sided) triangles which produces high global accuracy instead of resorting to quadratic curve triangles. The effectiveness of the present scheme has been checked by computing the modes and correspondent birefringency of elliptical core fibers reported in the literature. Also, a symmetric 2×2 fused-fiber coupler is analysed pointing out critical geometric parameters in the design of this structure.

Keywords: Finite Elements, Optical Curved Structures, Optical Fused Couplers, Optical Fibers, Integrated Optics.

1. INTRODUCTION

It is well known that smooth curved interfaces demand a special treatment in the analysis of optical waveguides. Typical examples arise in fibre technology, where curved cross-sections such as ellipses, and other complicated shapes may be fabricated or may be induced due to external effects - stress, for instance. See references [1], [2], and references therein. On the other hand, in integrated optics waveguide technology curved interfaces have not been yet properly exploited. Thus, a robust and easy way to handle this situation would be quite valuable to exploit curved dielectric domains in the design of integrated optics devices.

Although finite difference codes for handling this problem have been reported in the literature [4], this method is not as flexible as finite elements to deal with curved interfaces of arbitrary shape. In the finite-element optical-modelling literature, this problem has been treated by using meshes mainly composed of triangles (with straight sides) specially tailored to fit particular kinds of curved interfaces (circles and ellipses mainly) [1]. These meshes are generated in general without using an automatic and localized refinement at the interfaces. However, substantial improvement in the accuracy of that approach has been reported by using second-order triangular elements with curved (quadratic polynomials) sides, see Refs. [2] and [3].

The mesh used in this paper is adaptive and is based on a bisection algorithm [5], however, other algorithms may also be used [6]. Starting from a very coarse mesh and giving a set of suitable shape functions to monitor the refinement, a final refined mesh is readily obtained. The definition of those shape functions obey certain criteria established by the user. A typical criterion is to increase the density of triangles (finite elements) in regions where the user can expect more field intensity. When curved interfaces are included, one can also ask to increase the number of finite elements at such interfaces. Note that all this can be done locally, i. e. without affecting the density of other regions. Fig. 1 illustrates the present procedure for a fibre with elliptical cross-section (a quarter is shown). In this way the curved interface is fitted to a high degree of (local) accuracy, even using ordinary first order (straight sided) triangles.

2. RESULTS

2.1. Finite Element Numerical Implementation.

We used in this paper the scalar wave equation for an eletric field tranverse component E :

$$\partial^2 E/\partial x^2 + \partial^2 E/\partial y^2 + n^2 k_o^2 E = 0 \quad (1)$$

where $n = n(x, y)$ is the refractive index, which can be complex or not, and $k_o = \omega / c$. Assuming the modal exponential form $E(x, y) = \exp(jk_o \beta) u(x, y)$, where β is the fase constant, the equation (1) becomes :

$$\nabla_t^2 u + k_o^2 n^2 = k_o^2 \beta^2 u \quad (2)$$

By using the Finite Element Method applied to the Galerkin Functional, the equation (2) is transformed to an eigenvalue problem of the type :

$$Ax = \lambda Bx \quad (3)$$

In (3) the matrices have the form :

$$(A)_{i,j} = -\int_\Omega (\nabla\varphi_i.\nabla\varphi_j + k_o^2 n^2 \varphi_i.\varphi_j) d\Omega + \int_{\partial\Omega} (\partial\varphi_i / \partial\vartheta).\varphi_j dl,$$

and

$$(B)_{i,j} = k_o^2 \int_\Omega \varphi_i.\varphi_j d\Omega$$

where the matrices A and B are sparse, B is symmetric, $\lambda = \beta^2$, Ω represents the domain where the guided region is contained, $\partial\Omega$ includes all materials interfaces in Ω and represents an artificial closed curve chosen to separate the guiding (interior) region from the infinitely extended (exterior) region, and ϑ is the unit normal vector of the boundary $\partial\Omega$.

To analyse the behaviour of the field close to the boundary interfaces we adopt in this paper that the finite element approach for the modal analysis of the field has asymptotic behaviour and that its propagation taken place in open-boundary waveguides.

For convenience in this analysis we assume the harmonic time dependence $E(x, y, z) = \exp^{(-jk_o\gamma.z)} e(x, y)$, where γ is the normalised propagation constant, which may be real or complex.

We can also assume the radiation conditions such that the electromagnetic fields exhibit the following asymptotic form :

$$E = e(k\rho)[exp(-jk_\rho \rho)] / \sqrt{k_\rho} + O(1/\sqrt{(k_\rho \rho)^3})$$

where $\rho = \sqrt{(x^2 + y^2)}$, $k_\rho = k_o \sqrt{n^2 - \gamma^2}$ is the radial wavenumber, and n is the refractive index of the infinitely extended media outside some fictitious enclosing boundary.

Without loss of generality we can assume the media are linear and isotropic, and the origin of the coordinate system is located inside the core of the waveguide, or inside a region where the guided electromagnetic power is highly concentrated.

In this case, for the presence of the infinitely extended region, the finite element approach adopted to solve the guiding region can be taken into account by a line integral term over a fictitious boundary. Normally the integrand of this integral contains terms which are proportional to the tangential or normal derivates of the electromagnetic field components.

With our previous assumptions, those derivates can be expressed in terms of the of the field components themselves, and so we derivate the general impedance or mixed bounds conditions :

$$(\partial E / \partial\vartheta) = -jk_\rho \rho e(k\rho) \hat{u}_\rho . \hat{u}_\vartheta + O(1/\sqrt{(k_\rho \rho)^3}) \quad (4)$$

where $\hat{u}_\rho$ is the unit radial vector in cylindrical coordinates and $\hat{u}_\vartheta$ represents the unit normal vector related to the fictitious boundary interface.

If we substitute the equation (4) into the line integrals we derivate a nonlinear matrix eigenvalue problem of the form :

$$A(\gamma^2)x = \gamma^2 B(\gamma^2)x \quad (5)$$

The equation (5) can be solved iteratively in a self-consistent way. By using our iterative numerical process it is possible to calculate only one mode at a time. Besides the local nature of this approach, we can preserve the sparse form of the matrices involved in the equation (5), where generally the matrices A and B may be complex (non-Hermitian).

We used the subspace iteration method to solve (5), which takes full advantage of the sparsity of the matrices involved and allows one to calculate a given number of selected eigenvalues (and respective eigenvectors).

2.2. Elliptical Core Fibres.

In our first application, we illustrated the present procedure in Figure 1 for a fibre with elliptical cross-section (a quarter is shown). In this way the curved interface is fitted to a high degree of (local) accuracy, even using ordinary first order (straight sided) triangles.

We adopted three elliptical core fibres with aspect ratio e = 2, 3 and 5 ($e = a_x / a_y$), where a_x and a_y represent the corresponding axis. In all three fibres the refractive indices of core and cladding regions are n_1 =1.485 and n_2 =1.47, and the value of the semiminor axis a_y is 0.5 mm.

We define v = $k_o \; a_y \; ((n_1)^2 - (n_2)^2)^{0.5}$ and b = $((n_{eff})^2 - (n_2)^2)/((n_1)^2 - (n_2)^2)$, where n_{eff} is the effective refractive indice.

In Figure 2 we present the dispersion characteristics curves for the first, second and third TE and TM modes. The results corresponding to e = 2, 3 and 5, are shown in Figs. 2(a), 2(b) and 2(c), respectively. The normalizated polarization modal birefringence **B** curves are shown in Figure 3, where **B** is proportional to the difference between the TE and TM effective refractive indices. The results related to e = 2, 3 and 5, are depicted in Figs. 3(a), 3(b) and 3(c), respectively. Our numerical window was 12 μm × 8 μm, and the average number of unknows was 350.

Although our results show a good overall agreement with the ones reported in [2], in the cut-off regions, where more precision is required, our results exhibit a more accurate behaviour; i.e. asymptotic to the **v** axis.

The present strategy allows us to achieve more accurate results than those reported in [2], and this can be better appreciated from the birefingence results, which demand a precision of at least 5 decimals from the dispersion curves. It is noticiable that in our calculations we use ordinary first-order (straight sided) triangular elements, against the more complex second-order curved triangles reported in [2]. Although the number of elements we used are roughly 20 % greater than the one used in [2], taking into account that our resultant matrices are much sparser, our approach should consume considerable less time.

2.3. 2x2 Fused Fiber Couplers.

An accurate description of the wave propagation through a symmetric 2×2 fused fiber coupler can be obtained by using the coupled mode equation (CME) theory [10]. This requires the computation of the supermodes locally associated to the axially varying cross-section of this structure along the fused-taper or power exchange region. The shape of the two identical individalfibers, separated in the entrance and exit of the fused taper, change drastically along this coupling region. In [10] a realistic and careful model of the geometry of this kind of fibre coupler is reported, showing excellent agreement with experimental results. In that model, the local region where the maximum coupling of modes happen, i. e. the middle of the taper, the fused cladding is discribed as a circle. The two cores, which are assumed to keep their circular shape along the fusion process, are symmetrically placed inside the circular fused cladding and its centres are separated a distance $d = 2(2^{1/2} - 1)r_{cl}$, where r_{cl} is the unfused fiber-cladding radius.

Here, we investigate the effect of the fused cladding shape in the coupler performance, the fact of the fused cladding being elliptical rather than circular. We accomplish the present investigation by computing the associated supermodes using a scalar finite element formulation in conjunction with the efficient radiation boundary condition reported in [7], [8] and the adaptive refinement approach above.

We consider a symmetric 2×2 fused coupler made of identical fibres with a taper parameter τ=0.3. This parameter is described in [10]. The untapered fiber core and cladding radii are r_{co} = 2.25 μm and r_{cl} = 31.25 μm, respectively. The indices being n_{co} = 1.451813 and n_{cl} = 1.447313, respectively, and the wavelength λ = 1.3 mm. Figure 4(a) shows the initial refinement and numerical window used for a cross-section, with aspect ratio, e = 0.8, where $e = R_x / R_y$, and R_x and R_y represent the axis of the elliptical fused cladding. The final grid is presented in Figure 4(b), and we keep $R_y = 2^{1/2} r_{cl}$, constant along our calculations.

In Fig. 5 we show the curves of birefingence, **B** (×10^{-6}), for the LP_{01} and LP_{11} modes. Also, in the same figure the LP_{01} - LP_{11} beat length l (in cm), versus e, is shown. From these curves, the influence of the cladding shape in the design of this structure becomes clear. We observe that **B** varies significantly for LP_{11}, when e < 1. On the other hand, there is a compromise between **B** and l, namely, for e < 1, **B** becomes greater and l shorter. The opposite happens for e > 1.

Finally, Figures 6(a) and 6(b) show the LP_{01} and LP_{11} modal fields of a strongly fused coupler made of identical fibers.