Supplementary File

Title: Theoretical Outage Analysis of Nonlinear OFDM Systems with AF Relays

DETAILED DERIVATION OF (5), (6) AND (7)

Starting from (4) of the manuscript, we can write:

$$\begin{aligned}\gamma_n &= \frac{P_s}{\det \mathbf{R}_{\nu_\mathbf{n}}} \begin{bmatrix} h_{1n}^* & h_{2n}^* \end{bmatrix} \begin{bmatrix} \sigma^2_{\nu_{2n}} & -\rho_{\nu_{1n},\nu_{2n}} \\ -\rho^*_{\nu_{1n},\nu_{2n}} & \sigma^2_{\nu_{1n}} \end{bmatrix} \begin{bmatrix} h_{1n} \\ h_{2n} \end{bmatrix} \\ &= \frac{P_s}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left(|h_{1n}|^2 \sigma^2_{\nu_{2n}} - 2\mathrm{Re}[h_{1n} h_{2n}^* \rho^*_{\nu_{1n},\nu_{2n}}] + |h_{2n}|^2 \sigma^2_{\nu_{1n}} \right) \\ &= \frac{P_s |\alpha^{(S)}|^2}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left[|h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n|^2 \left(|h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} + \sigma^2_\eta \right) \right. \\ &\quad \left. -2\mathrm{Re}\left[h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n h_n^{(SD)*} h_n^{(SD)} h_n^{(SR)*} h_n^{(RD)*} g_n \alpha^{(R)*} \sigma^2_{d^{(S)}} \right] + |h_n^{(SD)}|^2 \sigma^2_{\nu_{1n}} \right], \quad \text{(A1)}\end{aligned}$$

where we used the following definitions provided in the manuscript: $h_{1n} = h_n^{(RD)} h_n^{(SR)} \alpha^{(S)} \alpha^{(R)} g_n$, $h_{2n} = h_n^{(SD)} \alpha^{(S)}$, $\nu_{1n} = h_n^{(SR)} h_n^{(RD)} \alpha^{(R)} g_n d_n^{(S)} + h_n^{(RD)} \alpha^{(R)} g_n \eta_n^{(SR)} + h_n^{(RD)} d_n^{(R)} + \eta_n^{(RD)}$ e $\nu_{2n} = h_n^{(SD)} d_n^{(S)} + \eta_n^{(SD)}$.

(A1) can be reexpressed as:

$$\begin{aligned}\gamma_n &= \frac{P_s |\alpha^{(S)}|^2}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left[|h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n|^2 \left(-|h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} + \sigma^2_\eta \right) + |h_n^{(SD)}|^2 \sigma^2_{\nu_{1n}} \right] \\ &= \frac{P_s |\alpha^{(S)}|^2}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left[|h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n|^2 \left(-|h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} + \sigma^2_\eta \right) \right. \\ &\quad \left. + |h_n^{(SD)}|^2 \left(|h_n^{(SR)}|^2 |h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_{d^{(S)}} + |h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_\eta + |h_n^{(RD)}|^2 \sigma^2_{d^{(R)}} + \sigma^2_\eta \right) \right] \\ &= \frac{P_s |\alpha^{(S)}|^2}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left[|h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n|^2 \sigma^2_\eta + |h_n^{(SD)}|^2 \left(|h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_\eta + |h_n^{(RD)}|^2 \sigma^2_{d^{(R)}} + \sigma^2_\eta \right) \right] \\ &= \frac{P_s |\alpha^{(S)}|^2}{\det \mathbf{R}_{\nu_\mathbf{n}}} \left[|h_n^{(RD)} h_n^{(SR)} \alpha^{(R)} g_n|^2 \sigma^2_\eta + |h_n^{(RD)} h_n^{(SD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_\eta \right. \\ &\quad \left. + |h_n^{(RD)} h_n^{(SD)}|^2 \sigma^2_{d^{(R)}} + |h_n^{(SD)}|^2 \sigma^2_\eta \right], \quad \text{(A2)}\end{aligned}$$

where:

$$\begin{aligned}\det \mathbf{R}_{\nu_\mathbf{n}} &= |h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_\eta |h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} + |h_n^{(RD)}|^2 \sigma^2_{d^{(R)}} |h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} + \sigma^2_\eta |h_n^{(SD)}|^2 \sigma^2_{d^{(S)}} \\ &\quad + |h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^4_\eta + |h_n^{(RD)}|^2 \sigma^2_{d^{(R)}} \sigma^2_\eta + \sigma^4_\eta + |h_n^{(SR)}|^2 |h_n^{(RD)}|^2 |\alpha^{(R)}|^2 g_n^2 \sigma^2_{d^{(S)}} \sigma^2_\eta. \end{aligned} \quad \text{(A3)}$$

By defining the following instantaneous SNRs of the wireless links:

$$\gamma_n^{(SD)} = \frac{\left| h_n^{(SD)} \alpha^{(S)} \right|^2 P_s}{\sigma^2_\eta}, \quad \text{(A4)}$$

$$\gamma_n^{(SR)} = \frac{\left| h_n^{(SR)} \alpha^{(S)} \right|^2 P_s}{\sigma_\eta^2} \tag{A5}$$

and

$$\gamma_n^{(RD)} = \frac{\left| h_n^{(RD)} \alpha^{(R)} \right|^2 P_r}{\sigma_\eta^2}, \tag{A6}$$

(A2) can be rewritten as

$$\gamma_n = \frac{\gamma_n^{num}}{\gamma_n^{dem}}, \tag{A7}$$

where

$$\begin{aligned}
\gamma_n^{num} &= P_s|\alpha^{(S)}|^2|h_n^{(RD)}|^2|h_n^{(SR)}\alpha^{(R)}g_n|^2\sigma_\eta^2 + P_s|\alpha^{(S)}|^2|h_n^{(RD)}|^2|h_n^{(SD)}|^2|\alpha^{(R)}|^2 g_n^2\sigma_\eta^2 \\
&\quad + P_s|\alpha^{(S)}|^2|h_n^{(RD)}|^2|h_n^{(SD)}|^2\sigma_{d^{(R)}}^2 + P_s|\alpha^{(S)}|^2|h_n^{(SD)}|^2\sigma_\eta^2 \\
&= \left(\gamma_n^{(RD)}\frac{\sigma_\eta^2}{P_r}\right)\left(\gamma_n^{(SR)}\sigma_\eta^2\right) g_n^2\sigma_\eta^2 + \left(\gamma_n^{(RD)}\frac{\sigma_\eta^2}{P_r}\right)\left(\gamma_n^{(SD)}\sigma_\eta^2\right) g_n^2\sigma_\eta^2 \\
&\quad + \left(\gamma_n^{(SD)}\sigma_\eta^2\right)\left(\gamma_n^{(RD)}\frac{\sigma_\eta^2}{P_r\left|\alpha^{(R)}\right|^2}\right)\sigma_{d^{(R)}}^2 + \left(\gamma_n^{(SD)}\sigma_\eta^2\right)\sigma_\eta^2 \\
&= \frac{\sigma_\eta^4 g_n^2}{P_r}\left[\gamma_n^{(RD)}\gamma_n^{(SR)}\sigma_\eta^2 + \gamma_n^{(RD)}\gamma_n^{(SD)}\sigma_\eta^2 + \frac{\gamma_n^{(SD)}\gamma_n^{(RD)}\sigma_{d^{(R)}}^2}{\left|\alpha^{(R)}\right|^2 g_n^2} + \frac{\gamma_n^{(SD)}P_r}{g_n^2}\right]
\end{aligned} \tag{A8}$$

and $\gamma_n^{den} = \det \mathbf{R}_{\nu_\mathbf{n}}$.

By defining

$$\gamma^{(PA_S)} = \frac{\left|\alpha^{(S)}\right|^2 P_s}{\sigma_{d^{(S)}}^2} \tag{A9}$$

and

$$\gamma^{(PA_R)} = \frac{\left|\alpha^{(R)}\right|^2 P_r}{\sigma_{d^{(R)}}^2}, \tag{A10}$$

the relay gain can be written as:

$$\begin{aligned} g_n^2 &= \frac{P_r}{\left|h_n^{(SR)}\alpha^{(S)}\right|^2 P_s + \left|h_n^{(SR)}\right|^2 \sigma_{d^{(S)}}^2 + \sigma_\eta^2} \\ &= \frac{P_r}{\gamma_n^{(SR)}\sigma_\eta^2 + \frac{\gamma_n^{(SR)}\sigma_\eta^2\sigma_{d^{(S)}}^2}{P_s|\alpha^{(S)}|^2} + \sigma_\eta^2} \\ &= \frac{P_r/\sigma_\eta^2}{\gamma_n^{(SR)} + \gamma_n^{(SR)}/\gamma^{(PA_S)} + 1} \\ &= \frac{\gamma^{(PA_S)} P_r/\sigma_\eta^2}{\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}, \end{aligned} \tag{A11}$$

which leads to

$$\frac{P_r}{g_n^2} = \sigma_\eta^2 \frac{\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}}. \tag{A12}$$

From (A9), (A10) and (A12), (A8) can be expressed by:

$$\begin{aligned} \gamma_n^{num} &= \frac{\sigma_\eta^4 g_n^2}{P_r}\left[\gamma_n^{(RD)}\gamma_n^{(SR)}\sigma_\eta^2 + \gamma_n^{(RD)}\gamma_n^{(SD)}\sigma_\eta^2 + \frac{\gamma_n^{(SD)}\gamma_n^{(RD)}}{\gamma^{(PA_R)}}\sigma_\eta^2\frac{\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}}\right. \\ &\left. +\gamma_n^{(SD)}\sigma_\eta^2\frac{\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}}\right] \\ &= \frac{\sigma_\eta^4 g_n^2}{\gamma^{(PA_S)}\gamma^{(PA_R)}P_r}\left[\gamma_n^{(RD)}\gamma_n^{(SR)}\gamma^{(PA_S)}\gamma^{(PA_R)}\sigma_\eta^2 + \gamma_n^{(RD)}\gamma_n^{(SD)}\gamma^{(PA_S)}\gamma^{(PA_R)}\sigma_\eta^2\right. \\ &+\gamma_n^{(SD)}\gamma_n^{(RD)}\sigma_\eta^2\left(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}\right) \\ &\left.+\gamma_n^{(SD)}\left(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}\right)\gamma^{(PA_R)}\sigma_\eta^2\right], \end{aligned} \tag{A13}$$

or:

$$\begin{aligned} \gamma_n^{num} &= \frac{\sigma_\eta^6 g_n^2}{\gamma^{(PA_S)}\gamma^{(PA_R)}P_r}\left[\gamma_n^{(RD)}\gamma_n^{(SR)}\gamma^{(PA_S)}\gamma^{(PA_R)} + \gamma_n^{(RD)}\gamma_n^{(SD)}\gamma^{(PA_S)}\gamma^{(PA_R)}\right. \\ &+\gamma_n^{(SD)}\gamma_n^{(RD)}\left(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}\right) \\ &\left.+\gamma_n^{(SD)}\gamma^{(PA_R)}\left(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}\right)\right] \\ &= \frac{\sigma_\eta^6 g_n^2}{\gamma^{(PA_S)2}\gamma^{(PA_R)}P_r}\left[\gamma_n^{(RD)}\gamma_n^{(SR)}\gamma_n{}^{(PA_S)2}\gamma^{(PA_R)} + \gamma_n^{(RD)}\gamma_n^{(SD)}\gamma_n{}^{(PA_S)2}\gamma^{(PA_R)}\right. \\ &+\gamma_n^{(SD)}\gamma_n^{(RD)}\gamma^{(PA_S)}(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}) \\ &\left.+\gamma_n^{(SD)}\gamma^{(PA_S)}\gamma^{(PA_R)}(\gamma_n^{(SR)}\gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)})\right]. \end{aligned} \tag{A14}$$

On the other hand, (A3) can be rewritten as follows:

$$\begin{aligned}
\gamma_n^{den} = \det \mathbf{R}_{\nu_\mathbf{n}} \quad = \quad & \left(\gamma_n^{(RD)} \frac{\sigma_\eta^2}{P_r}\right) g_n^2 \sigma_\eta^2 \left(\gamma_n^{(SD)} \frac{\sigma_\eta^2}{P_s \left|\alpha^{(S)}\right|^2}\right) \sigma_{d^{(S)}}^2 \\
& + \left(\gamma_n^{(RD)} \frac{\sigma_\eta^2}{P_r \left|\alpha^{(R)}\right|^2}\right) \sigma_{d^{(R)}}^2 \left(\gamma_n^{(SD)} \frac{\sigma_\eta^2}{P_s \left|\alpha^{(S)}\right|^2}\right) \sigma_{d^{(S)}}^2 + \left(\gamma_n^{(SD)} \frac{\sigma_\eta^2}{P_s \left|\alpha^{(S)}\right|^2}\right) \sigma_\eta^2 \sigma_{d^{(S)}}^2 \\
& + \left(\gamma_n^{(RD)} \frac{\sigma_\eta^2}{P_r}\right) g_n^2 \sigma_\eta^4 + \left(\gamma_n^{(RD)} \frac{\sigma_\eta^2}{P_r \left|\alpha^{(R)}\right|^2}\right) \sigma_{d^{(R)}}^2 \sigma_\eta^2 + \sigma_\eta^4 \\
& + \left(\gamma_n^{(SR)} \frac{\sigma_\eta^2}{P_s \left|\alpha^{(S)}\right|^2}\right) \left(\gamma_n^{(RD)} \frac{\sigma_\eta^2}{P_r}\right) g_n^2 \sigma_{d^{(S)}}^2 \sigma_\eta^2, \qquad \text{(A15)}
\end{aligned}$$

or, equivalently:

$$\begin{aligned}
\gamma_n^{den} \quad = \quad & \frac{\sigma_\eta^4 g_n^2}{P_r} \Bigg[\gamma_n^{(RD)} \sigma_\eta^2 \left(\gamma_n^{(SD)} \frac{1}{P_s \left|\alpha^{(S)}\right|^2}\right) \sigma_{d^{(S)}}^2 + \left(\gamma_n^{(RD)} \frac{1}{\left|\alpha^{(R)}\right|^2}\right) \sigma_{d^{(R)}}^2 \left(\gamma_n^{(SD)} \frac{1}{P_s \left|\alpha^{(S)}\right|^2}\right) \sigma_{d^{(S)}}^2 \frac{1}{g_n^2} \\
& + \left(\gamma_n^{(SD)} \frac{1}{P_s \left|\alpha^{(S)}\right|^2}\right) \frac{P_r}{g_n^2} \sigma_{d^{(S)}}^2 + \gamma_n^{(RD)} \sigma_\eta^2 + \frac{\gamma_n^{(RD)} \sigma_{d^{(R)}}^2}{\left|\alpha^{(R)}\right|^2 g_n^2} + \frac{P_r}{g_n^2} \\
& + \left(\gamma_n^{(SR)} \frac{1}{P_s \left|\alpha^{(S)}\right|^2}\right) \gamma_n^{(RD)} \sigma_{d^{(S)}}^2 \sigma_\eta^2 \Bigg] \\
= \quad & \frac{\sigma_\eta^4 g_n^2}{P_r} \Bigg[\frac{\gamma_n^{(RD)} \sigma_\eta^2 \gamma_n^{(SD)}}{\gamma^{(PA_S)}} + \frac{\gamma_n^{(RD)} \gamma_n^{(SD)} \sigma_\eta^2}{\gamma^{(PA_S)} \gamma^{(PA_R)}} \frac{\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}} \\
& + \frac{\gamma_n^{(SD)} \sigma_\eta^2}{\gamma^{(PA_S)}} \frac{\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}} + \gamma_n^{(RD)} \sigma_\eta^2 \\
& + \frac{\gamma_n^{(RD)} \sigma_\eta^2}{\gamma^{(PA_R)}} \frac{\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}} + \sigma_\eta^2 \frac{\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}} + \frac{\gamma_n^{(SR)} \gamma_n^{(RD)} \sigma_\eta^2}{\gamma^{(PA_S)}}. \qquad \text{(A16)}
\end{aligned}$$

(A16) can be reexpressed as:

$$\begin{aligned}
\gamma_n^{den} &= \frac{\sigma_\eta^4 g_n^2}{P_r \gamma^{(PA_S)}} \Bigg[\gamma_n^{(RD)} \sigma_\eta^2 \gamma_n^{(SD)} + \frac{\gamma_n^{(RD)} \gamma_n^{(SD)} \sigma_\eta^2}{\gamma^{(PA_R)}} \frac{\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)}}{\gamma^{(PA_S)}} \\
&\quad + \frac{\gamma_n^{(SD)} \sigma_\eta^2 \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right)}{\gamma^{(PA_S)}} + \gamma_n^{(RD)} \gamma^{(PA_S)} \sigma_\eta^2 \\
&\quad + \frac{\gamma_n^{(RD)} \sigma_\eta^2}{\gamma^{(PA_R)}} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) \\
&\quad + \sigma_\eta^2 \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) + \gamma_n^{(SR)} \gamma_n^{(RD)} \sigma_\eta^2 \Bigg] \\
&= \frac{\sigma_\eta^6 g_n^2}{P_r \gamma^{(PA_S)} \gamma^{(PA_R)}} \Bigg[\gamma_n^{(RD)} \gamma_n^{(SD)} \gamma^{(PA_R)} + \frac{\gamma_n^{(RD)} \gamma_n^{(SD)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right)}{\gamma^{(PA_S)}} \\
&\quad + \frac{\gamma_n^{(SD)} \gamma^{(PA_R)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right)}{\gamma^{(PA_S)}} + \gamma_n^{(RD)} \gamma_n^{(PA_s)} \gamma^{(PA_R)} \\
&\quad + \gamma_n^{(RD)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) + \gamma^{(PA_R)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) \\
&\quad + \gamma_n^{(SR)} \gamma_n^{(RD)} \gamma^{(PA_R)} \Bigg], \qquad \text{(A17)}
\end{aligned}$$

which leads to:

$$\begin{aligned}
\gamma_n^{den} &= \frac{\sigma_\eta^6 g_n^2}{P_r {\gamma^{(PA_S)}}^2 \gamma^{(PA_R)}} \Big[\gamma_n^{(RD)} \gamma_n^{(SD)} \gamma^{(PA_R)} \gamma^{(PA_S)} + \gamma_n^{(RD)} \gamma_n^{(SD)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) \\
&\quad + \gamma_n^{(SD)} \gamma^{(PA_R)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) + \gamma_n^{(RD)} {\gamma^{(PA_S)}}^2 \gamma^{(PA_R)} \\
&\quad + \gamma_n^{(RD)} \gamma^{(PA_S)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) + \\
&\quad \gamma^{(PA_R)} \gamma^{(PA_S)} \left(\gamma_n^{(SR)} \gamma^{(PA_S)} + \gamma_n^{(SR)} + \gamma^{(PA_S)} \right) + \gamma_n^{(SR)} \gamma_n^{(RD)} \gamma^{(PA_R)} \gamma^{(PA_S)}. \qquad \text{(A18)}
\end{aligned}$$

(5), (6) and (7) of the manuscript are obtained from (A7), (A14) and (A18).